Guided Practice 1.1.1

Example 1

Reduce the radical expression $\sqrt{\frac{80}{5^4}}$. If the result has a root in the denominator, rationalize it. Is the result rational or irrational?

1. Rewrite each number in the expression as a product of prime numbers.

   The denominator of the expression under the radical sign, 5^4, is already written as a prime factorization. Rewrite the numerator, 80, as the product of its prime factors, then group identical factors together using exponents:

$\sqrt{\frac{80}{5^4}}$	Original expression
$\sqrt{\frac{2 \bullet 2 \bullet 2 \bullet 2 \bullet 5}{5^4}}$	Rewrite 80 as a product of its prime factors.
$\sqrt{\frac{2^4 \bullet 5}{5^4}}$	Group prime factors together with exponents.

2. Cancel where possible to reduce the resulting expression.

   Divide out factors that appear in both the numerator and the denominator.

   The expression $\sqrt{\dfrac{2^4 \bullet 5}{5^4}}$ has 5s in the numerator and denominator, so 5 is a common factor that will cancel out.

   Recall that when canceling terms in an expression that contains an exponent, you reduce the power of the exponent by 1 for each factor you cancel.

$\sqrt{\dfrac{2^4 \bullet 5}{5^4}}$	Expression from the previous step
$\sqrt{\dfrac{2^4 \bullet \cancel{5}}{5^{\cancel{4}\,3}}}$	Cancel out common factors.
$\sqrt{\dfrac{2^4}{5^3}}$	Reduced expression

   There are no more factors that appear in both the numerator and denominator, so nothing more can be cancelled out.

3. Use the properties of radicals to rewrite the reduced expression.

   Rewrite the expression as a fraction of radicals, and solve any squares under the radical sign.

$\sqrt{\dfrac{2^4}{5^3}}$	Expression from the previous step
$\dfrac{\sqrt{2^4}}{\sqrt{5^3}}$	Rewrite using the quotient property of radicals.
$\dfrac{\sqrt{2^4}}{\sqrt{5^2 \bullet 5}}$	Factor out the perfect square in the denominator.
$\dfrac{\sqrt{2^4}}{\sqrt{5^2} \bullet \sqrt{5}}$	Rewrite using the product property of radicals.
$\dfrac{2^2}{5\sqrt{5}}$	Evaluate the radicals.
$\dfrac{4}{5\sqrt{5}}$	Evaluate the exponent.

4. Rationalize the denominator of the resulting fraction.

   To rationalize the denominator, multiply both the numerator and the denominator by the radical in the denominator. This is equivalent to multiplying by 1, and thus does not change the value of the fraction.

$\dfrac{4}{5\sqrt{5}}$	Expression from the previous step
$\dfrac{4}{5\sqrt{5}} \bullet \dfrac{\sqrt{5}}{\sqrt{5}}$	Multiply the numerator and the denominator by the radical in the denominator.
$\dfrac{4 \bullet \sqrt{5}}{5 \bullet 5}$	Simplify.
$\dfrac{4\sqrt{5}}{25}$	Multiply.

   The expression $\sqrt{\dfrac{80}{5^4}}$ is equal to $\dfrac{4\sqrt{5}}{25}$.

5. Determine whether the resulting expression is rational or irrational.

   The expression $\dfrac{4\sqrt{5}}{25}$ cannot be written as a ratio of whole numbers and is therefore irrational.

Example 2

Reduce the radical expression $\sqrt{16a^2}+\sqrt{32a^4}$. Assuming a is a whole number, is the result rational or irrational?

1. Use the properties of radicals to rewrite the expression.

   Rewrite each radical in the expression as a product of radicals, and evaluate where possible.

$\sqrt{16a^2}+\sqrt{32a^4}$	Original expression
$\sqrt{16}\bullet\sqrt{a^2}+\sqrt{32}\bullet\sqrt{a^4}$	Rewrite using the product property of radicals.
$4\bullet a+\sqrt{32}\bullet a^2$	Evaluate the radical perfect squares, $\sqrt{16}$, $\sqrt{a^2}$, and $\sqrt{a^4}$.
$4a+\sqrt{32}\bullet a^2$	Simplify.

   The radical expression $\sqrt{16a^2}+\sqrt{32a^4}$ can be rewritten as $4a+\sqrt{32}\bullet a^2$.

2. Reduce any remaining radicals.

   We have one remaining radical, $\sqrt{32}$. Rewrite 32 as a product with a perfect square, and simplify using the properties of radicals.

$4a+\sqrt{32}\bullet a^2$	Simplified expression from the previous step
$4a+\sqrt{16\bullet 2}\bullet a^2$	Factor out the perfect square in the radicand.
$4a+\sqrt{16}\bullet\sqrt{2}\bullet a^2$	Rewrite using the product property of radicals.
$4a+4\bullet\sqrt{2}\bullet a^2$	Evaluate the radical perfect square, $\sqrt{16}$.
$4a+4\sqrt{2}\bullet a^2$	Simplify.

   The remaining radical, $\sqrt{2}$, cannot be further reduced. The final reduced expression is $4a+4\sqrt{2}\bullet a^2$.

3. Determine whether the resulting expression is rational or irrational.

   Because a is a whole number, the first part of the reduced expression, $4a$, is rational. The second part of the expression, $4\sqrt{2}\bullet a^2$, is a product of rational numbers and an irreducible radical. Therefore, the second part of the expression is irrational. Because the sum of a rational number and an irrational number is irrational, the entire expression must be irrational.

Example 3

Evaluate the radical expression $\sqrt{\frac{2^6}{45}}\left(\sqrt{\frac{64}{5^3}}+\sqrt{\frac{18}{250}}\right)$. Then, determine whether the answer is rational or irrational.

1. Rewrite each number in the expression as a product of prime numbers.

   Evaluating the expression will be much easier if we have lists of only prime factors to work with. Group identical factors together using exponents.

$\sqrt{\frac{2^6}{45}}\left(\sqrt{\frac{64}{5^3}}+\sqrt{\frac{18}{250}}\right)$	Original expression
$\sqrt{\frac{2^6}{3\bullet 3\bullet 5}}\left(\sqrt{\frac{2\bullet 2\bullet 2\bullet 2\bullet 2\bullet 2}{5^3}}+\sqrt{\frac{2\bullet 3\bullet 3}{2\bullet 5\bullet 5\bullet 5}}\right)$	Rewrite each composite number as a product of its prime factors.
$\sqrt{\frac{2^6}{3^2\bullet 5}}\left(\sqrt{\frac{2^6}{5^3}}+\sqrt{\frac{2\bullet 3^2}{2\bullet 5^3}}\right)$	Group identical factors using exponents.

2. Cancel where possible to reduce the resulting expression.

   Cancel any terms that appear in both the numerator and the denominator of each fraction.

   $$\sqrt{\frac{2^6}{3^2 \bullet 5}}\left(\sqrt{\frac{2^6}{5^3}}+\sqrt{\frac{2 \bullet 3^2}{2 \bullet 5^3}}\right)$$ Expression from the previous step

   $$\sqrt{\frac{2^6}{3^2 \bullet 5}}\left(\sqrt{\frac{2^6}{5^3}}+\sqrt{\frac{\cancel{2} \bullet 3^2}{\cancel{2} \bullet 5^3}}\right)$$ Cancel out common factors.

   $$\sqrt{\frac{2^6}{3^2 \bullet 5}}\left(\sqrt{\frac{2^6}{5^3}}+\sqrt{\frac{3^2}{5^3}}\right)$$ Reduced expression

   There are no more factors that appear in both the numerator and denominator, so nothing more can be cancelled out.

   Now that we have an expression written in terms of prime numbers only, we can move to the next step.

3. Distribute the radical outside the parentheses, and rewrite using the properties of radicals.

   Distribute the radical expression outside the parentheses among the expressions inside the parentheses. Then, reduce the result using the properties of radicals.

$\sqrt{\dfrac{2^6}{3^2 \bullet 5}}\left(\sqrt{\dfrac{2^6}{5^3}}+\sqrt{\dfrac{3^2}{5^3}}\right)$	Simplified expression from the previous step
$\sqrt{\dfrac{2^6}{3^2 \bullet 5}} \bullet \sqrt{\dfrac{2^6}{5^3}}+\sqrt{\dfrac{2^6}{3^2 \bullet 5}} \bullet \sqrt{\dfrac{3^2}{5^3}}$	Distribute the outer radical expression.
$\sqrt{\dfrac{2^6 \bullet 2^6}{3^2 \bullet 5 \bullet 5^3}}+\sqrt{\dfrac{2^6 \bullet 3^2}{3^2 \bullet 5 \bullet 5^3}}$	Rewrite using the product property of radicals.
$\sqrt{\dfrac{2^{12}}{3^2 \bullet 5^4}}+\sqrt{\dfrac{2^6 \bullet 3^2}{3^2 \bullet 5^4}}$	Combine identical factors using exponents.
$\sqrt{\dfrac{2^{12}}{3^2 \bullet 5^4}}+\sqrt{\dfrac{2^6 \bullet \cancel{3^2}}{\cancel{3^2} \bullet 5^4}}$	Cancel out common factors.
$\sqrt{\dfrac{2^{12}}{3^2 \bullet 5^4}}+\sqrt{\dfrac{2^6}{5^4}}$	Reduced expression

4. Use the properties of radicals to rewrite the reduced expression.

   The next step is to reduce the radical fractions to a fraction of radicals. After that, we can further reduce the result to determine the final value of the reduced expression.

$\sqrt{\frac{2^{12}}{3^2 \bullet 5^4}}+\sqrt{\frac{2^6}{5^4}}$	Expression from the previous step
$\frac{\sqrt{2^{12}}}{\sqrt{3^2 \bullet 5^4}}+\frac{\sqrt{2^6}}{\sqrt{5^4}}$	Rewrite using the quotient property of radicals.
$\frac{\sqrt{2^{12}}}{\sqrt{3^2} \bullet \sqrt{5^4}}+\frac{\sqrt{2^6}}{\sqrt{5^4}}$	Rewrite using the product property of radicals.
$\frac{2^6}{3 \bullet 5^2}+\frac{2^3}{5^2}$	Evaluate the radical perfect squares.
$\frac{2^6}{3 \bullet 5^2}+\frac{3 \bullet 2^3}{3 \bullet 5^2}$	Rewrite $\frac{2^3}{5^2}$ so it has a denominator of $3 \bullet 5^2$ by multiplying the numerator and denominator by 3.
$\frac{2^6+3 \bullet 2^3}{3 \bullet 5^2}$	Combine the fractions.
$\frac{64+24}{3 \bullet 5^2}$	Simplify the expressions in the numerator.

(continued)

$\frac{88}{3 \bullet 5^2}$	Add the expressions in the numerator.
$\frac{88}{75}$	Simplify the expressions in the denominator.

The expression $\sqrt{\frac{2^6}{45}}\left(\sqrt{\frac{64}{5^3}}+\sqrt{\frac{18}{250}}\right)$ simplifies to $\frac{88}{75}$.

5. Determine whether the resulting expression is rational or irrational.

   The expression can be written as a ratio of two whole numbers, so it is rational.

Example 4

Professor Oak is building a new paddock in the back of his research facility so his pets can stay outside while he's at work. According to his calculations, the amount of fencing required will be $2\sqrt{4800}+\left(160-8\sqrt{300}\right)$ feet. If fencing is sold in 5-foot lengths, how many pieces of fencing will he need to purchase to complete the paddock?

1. Reduce the expression using the properties of radicals.

   Use the properties for rewriting radicals to simplify the expression.

$2\sqrt{4800}+\left(160-8\sqrt{300}\right)$	Original expression
$2\sqrt{100\bullet16\bullet3}+160-8\sqrt{100\bullet3}$	Factor out perfect squares in the radicands.
$2\bullet\sqrt{100}\bullet\sqrt{16}\bullet\sqrt{3}+160-8\bullet\sqrt{100}\bullet\sqrt{3}$	Rewrite using the product property of radicals.
$2\bullet10\bullet4\bullet\sqrt{3}+160-8\bullet10\bullet\sqrt{3}$	Evaluate the radical perfect squares.
$80\sqrt{3}+160-80\sqrt{3}$	Simplify.
$\cancel{80\sqrt{3}}+160-\cancel{80\sqrt{3}}$	Add like terms.
160	

   Professor Oak requires 160 feet of fencing to complete the new paddock.

2. Determine the number of fencing units Professor Oak needs to buy.

   Because the fencing is sold in 5-foot lengths, divide the total length of fencing required by 5 to find how many units Professor Oak needs to purchase.

   Professor Oak needs 160 feet of fencing, so he will need to purchase $160 \div 5 = 32$ units of fencing.