Instruction

Problem-Based Task 1.1.1: Measuring Madness

Coaching Sample Responses

a. How can you find the diameter of each bowl?

To find the diameter of each bowl, substitute the given volume and height into the formula $d = 2\sqrt{\frac{V}{3h}}$, and simplify. To make it easier, assign a number to each bowl, as in the following table.

Bowl #	Volume (cm^3)	Height (cm)
1	480	6
2	240	5
3	120	4
4	80	3.5
5	60	3.5

b. How can you tell if a number is rational or irrational?

A rational number is a real number that has a terminating or repeating decimal, and can be written as the ratio of two integers; i.e., as a fraction. An irrational number has a decimal that does not end or repeat, and it cannot be written as a fraction.

c. What is the diameter of each bowl, and is the diameter rational or irrational?

Substitute the known dimensions for each bowl into the formula $d = 2\sqrt{\frac{V}{3h}}$ and solve.

For bowl 1:

$$d = 2\sqrt{\frac{V}{3h}}$$

$$d_1 = 2\sqrt{\frac{480}{3 \bullet 6}}$$

$$d_1 = 2\sqrt{\frac{\cancel{6} \cdot 80}{3 \cdot \cancel{6}}}$$

$$d_1 = 2\sqrt{\frac{80}{3}}$$

$$d_1 = 2\sqrt{\frac{16 \cdot 5}{3}}$$

$$d_1 = 2 \cdot 4\sqrt{\frac{5}{3}}$$

$$d_1 = 8\sqrt{\frac{5}{3}} \approx 10.3$$

Bowl 1 has a diameter of $8\sqrt{\frac{5}{3}}$ cm, or approximately 10.3 cm. The decimal never ends or repeats, so this bowl's diameter is irrational.

For bowl 2:

$$d = 2\sqrt{\frac{V}{3h}}$$

$$d_2 = 2\sqrt{\frac{240}{3 \cdot 5}}$$

$$d_2 = 2\sqrt{\frac{\cancel{3} \cdot \cancel{5} \cdot 16}{\cancel{3} \cdot \cancel{5}}}$$

$$d_2 = 2\sqrt{\frac{16}{1}}$$

$$d_2 = 2 \cdot 4$$

$$d_2 = 8$$

Bowl 2 has a diameter of exactly 8 cm, so this bowl's diameter is rational.

For bowl 3:

$$d = 2\sqrt{\frac{V}{3h}}$$

$$d_3 = 2\sqrt{\frac{120}{3 \bullet 4}}$$

$$d_3 = 2\sqrt{\frac{\cancel{3} \bullet \cancel{4} \bullet 10}{\cancel{3} \bullet \cancel{4}}}$$

$$d_3 = 2\sqrt{\frac{10}{1}}$$

$$d_3 = 2\sqrt{10} \approx 6.3$$

Bowl 3 has a diameter of $2\sqrt{10}$ cm, or approximately 6.3 cm. The decimal never ends or repeats, so this bowl's diameter is irrational.

For bowl 4:

$$d = 2\sqrt{\frac{V}{3h}}$$

$$d_4 = 2\sqrt{\frac{80}{3 \bullet 3.5}}$$

$$d_4 = 2\sqrt{\frac{800}{3 \bullet 35}}$$

$$d_4 = 2\sqrt{\frac{\cancel{5} \bullet 16 \bullet 10}{3 \bullet \cancel{5} \bullet 7}}$$

$$d_4 = 2 \bullet 4\sqrt{\frac{10}{3 \bullet 7}}$$

$$d_4 = 8\sqrt{\frac{10}{21}} \approx 5.5$$

Bowl 4 has a diameter of $8\sqrt{\frac{10}{21}}$ cm, or approximately 5.5 cm. The decimal never ends or repeats, so this bowl's diameter is irrational.

For bowl 5:

$$d = 2\sqrt{\frac{V}{3h}}$$

$$d_5 = 2\sqrt{\frac{60}{3 \bullet 3.5}}$$

$$d_5 = 2\sqrt{\frac{600}{3 \bullet 35}}$$

$$d_5 = 2\sqrt{\frac{\cancel{3} \bullet \cancel{5} \bullet 4 \bullet 10}{\cancel{3} \bullet \cancel{5} \bullet 7}}$$

$$d_5 = 2 \bullet 2\sqrt{\frac{10}{7}}$$

$$d_5 = 4\sqrt{\frac{10}{7}} \approx 4.8$$

Bowl 5 has a diameter of $4\sqrt{\frac{10}{7}}$ cm, or approximately 4.8 cm. The decimal never ends or repeats, so this bowl's diameter is irrational.

d. How can you figure out whether the bowls will fit on the shelf?

Roxanne wants to display the bowls in a line on a shelf that is 45 cm long and 15 cm wide. The bowls will fit on the shelf if two conditions are met: no bowl can be wider than the shelf, and the length of the line must be shorter than the shelf. In other words, the bowls will fit if the sum of the diameters is less than 45 cm and each diameter is less than 15 cm.

In part c, we calculated the diameter for each bowl. Every diameter was less than 15 cm, so now we must check that the sum of the diameters is less than 45 cm.

e. What is the sum of the diameters of the bowls, and is the sum rational or irrational?

Add the diameters of the bowls together, and simplify if possible.

$$\text{Line length} = d_1 + d_2 + d_3 + d_4 + d_5$$

$$\text{Line length} = 8\sqrt{\frac{5}{3}} + 8 + 2\sqrt{10} + 8\sqrt{\frac{10}{21}} + 4\sqrt{\frac{10}{7}} \approx 35 \text{ cm}$$

The approximate length of the line of bowls is 35 cm. The expression contains only irreducible radicals. Therefore, this quantity is irrational because it is a sum of rational and irrational numbers.

f. Will the bowls fit on the shelf in a line? Why or not?

The bowls will fit on the shelf in a line. No bowl is wider than the shelf, and the combined length of the diameters of the bowls is less than the length of the shelf.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.