## UNIT 1 • RELATIONSHIIPS BETWEEN QUANTITIES AND EXPRESSIONS

Lesson 2: Units of Measure

## Instruction

## Guided Practice 1.2.1

## Example 1

Kolya wants to find out how many textbooks his class has in total. He has discovered that each student has an average of 5 textbooks. There are 30 people in his class. How many textbooks does his class have in total?

1. Identify important quantities and their associated units.

We are given that each student has an average of 5 textbooks, and that there are 30 students in the class. Rewriting with units, we get the ratios $\frac{5 \text { textbooks }}{1 \text { student }}$ and $\frac{30 \text { students }}{1 \text { class }}$.
2. Identify the units of the answer being sought.

Kolya wants to find how many textbooks his class has in total. In other words, he wants to find the number of textbooks per class: textbooks
class . Therefore, the units of the answer are "textbooks" and "class."

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3. Convert the units.

In step 1, we found that the first quantity, $\frac{5 \text { textbooks }}{1 \text { student }}$, has the unit "textbooks" in the numerator, while the second quantity, $\frac{30 \text { class }}{1}$, has the unit "class" in the denominator. Notice that both ratios include the unit "students": one has it in the numerator, and the other has it in the denominator. We want to use an arithmetic operation to convert these two quantities to a quantity with units of $\frac{\text { textbooks }}{\text { class }}$. To do this, we multiply the two quantities and cancel units that appear in both the numerator and denominator.


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## Instruction

## Example 2

How many seconds are there in 1 day?

1. Determine the target units.

We want to convert days to seconds. In other words, we need to
know how many seconds there are per day. The proper units will be seconds day
2. Determine necessary conversion factors.

To convert days to seconds, use your knowledge of time measurement to change the units. There are 60 seconds in a minute, 60 minutes in an hour, and 24 hours in a day. The conversion factors are therefore $\frac{60 \text { seconds }}{1 \text { minute }}, \frac{60 \text { minutes }}{1 \text { hour }}$, and $\frac{24 \text { hours }}{1 \text { day }}$.

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3. Multiply the conversion factors.

In the previous step, we found that we will use three conversion factors to change days to seconds. Set the conversion factors in a multiplication problem so the intermediate factor will cancel out unwanted units in the other two, then multiply the numbers. In this case, we just multiply all three factors together in order of increasing unit size.

| $\frac{60 \text { seconds }}{1 \text { minute }} \bullet \frac{60 \text { minutes }}{1 \text { hour }} \bullet \frac{24 \text { hours }}{1 \text { day }}$ | Set up the calculation. |
| :--- | :--- |
| $\frac{60 \text { seconds }}{1 \text { minute }} \bullet \frac{60 \text { minutes }}{1 \text { hour }} \bullet \frac{24 \text { hours }}{1 \text { day }}$ | Cancel units that appear <br> in the numerator and <br> denominator. <br> Simplify. |
| $\frac{60 \bullet 60 \bullet 24 \text { seconds }}{1 \text { day }}$ | Multiply. |
| $\frac{86,400 \text { seconds }}{1 \text { day }}$ |  |
| There are 86,400 seconds in 1 day. |  |

## UNIT 1 • RELATIONSHIPS BETWEEN QUANTITIES AND EXPRESSIONS

## Lesson 2: Units of Measure

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## Example 3

Gemma is visiting friends in the U.S. She wants to make her famous mince pies, but her recipe lists most of the ingredients in grams. Use the chart to convert all the given measurements to U.S. units.

## Ingredients:

225 grams cold butter, diced
340 grams plain flour
110 grams golden caster sugar
300 grams mincemeat
1 small egg
Conversion Factors

| U.S. | Metric |
| :---: | :---: |
| 1 stick butter | 113 grams |
| 1 cup | 225 grams |
| 1 pound | 455 grams |
| 1 teaspoon | 5 grams |

5 grams powdered sugar

1. Identify which units need to be converted.

All the ingredients except the egg are given in grams. Therefore, we will need to convert all the ingredients except the egg into U.S. measurements.
2. Identify the target units.

In U.S. units, butter is usually measured in sticks, flour and sugar are usually measured in cups, meat is usually measured in pounds or ounces, and small amounts are usually measured in teaspoons. So, we want to convert butter to sticks, flour and caster sugar to cups, mincemeat to pounds, and powdered sugar to teaspoons.
3. Set up the conversion and solve.

Use the list of conversion factors to set up the conversion. Because we are converting out of grams, the conversion factor should always have grams in the denominator and the target units in the numerator. The original units should be in terms of grams per recipe.
For butter:
$\frac{225 \text { grams }}{1 \text { recipe }} \cdot \frac{1 \text { stick }}{113 \text { grams }} \quad$ Set up the conversion.

$\frac{225 \text { gramss }}{1 \text { recipe }} \cdot \frac{1 \text { stick }}{113 \text { grams }} \quad$| Cancel units appearing in the |
| :--- |
| numerator and denominator. |

$\frac{225 \text { sticks }}{113 \text { recipe }} \approx \frac{2 \text { sticks }}{1 \text { recipe }} \quad$ Simplify.

This recipe calls for approximately 2 sticks of butter.

For flour:

| $\frac{340 \text { grams }}{1 \text { recipe }} \cdot \frac{1 \text { cup }}{225 \text { grams }}$ | Set up the conversion. |
| :--- | :--- |
| $\frac{340 \text { grams }}{1 \text { recipe }} \cdot \frac{1 \text { cup }}{225 \text { grams }}$ | Cancel units appearing in the <br> numerator and denominator. |
| $\frac{340 \text { cups }}{225 \text { recipes }} \approx \frac{1.51 \text { cups }}{1 \text { recipe }}$ | Simplify. |

This recipe calls for approximately $1 \frac{1}{2}$ cups of flour.
(continued)

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For sugar:


$\frac{110 \text { grams }}{1 \text { recipe }} \cdot \frac{1 \text { cup }}{225 \text { grams }} \quad$| Cancel units appearing in the |
| :--- |
| numerator and denominator. |

$\frac{110 \text { cups }}{225 \text { recipes }} \approx \frac{0.49 \text { cups }}{1 \text { recipe }} \quad$ Simplify.

This recipe calls for approximately $\frac{1}{2}$ cup of sugar.

For mincemeat:

| $\frac{300 \text { grams }}{1 \text { recipe }} \bullet \frac{1 \text { pound }}{455 \text { grams }}$ | Set up the conversion. |
| :--- | :--- |
| $\frac{300 \text { grams }}{1 \text { recipe }} \bullet \frac{1 \text { pound }}{455 \text { grams }}$ | Cancel units appearing in the <br> numerator and denominator. |
| $\frac{300 \text { pounds }}{455 \text { recipe }} \approx \frac{0.66 \text { pounds }}{1 \text { recipe }}$ | Simplify. |

This recipe calls for approximately $\frac{2}{3}$ pound of mincemeat.
(continued)

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For powdered sugar:
$\frac{5 \text { grams }}{1 \text { recipe }} \bullet \frac{1 \text { teaspoon }}{5 \text { grams }} \quad$ Set up the conversion.
$\frac{5 \text { grams }}{1 \text { recipe }} \bullet \frac{1 \text { teaspoon }}{5 \text { grams }} \quad \begin{aligned} & \text { Cancel units appearing in the } \\ & \text { numerator and denominator. }\end{aligned}$
$\frac{5 \text { teaspoons }}{5 \text { recipes }}=\frac{1 \text { teaspoon }}{1 \text { recipe }} \quad$ Simplify.

This recipe calls for 1 teaspoon of powdered sugar.
4. Use the conversions to rewrite the list of ingredients.

The ingredients list, in U.S. units, is as follows:
2 sticks cold butter, diced
$1 \frac{1}{2}$ cups plain flour
1
$\frac{1}{2}$ cup golden caster sugar
2
$\overline{3}$ pound mincemeat
1 small egg
1 teaspoon powdered sugar

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## Example 4

A company that makes electric heaters would like to upgrade its production equipment to become more profitable. The company hired two separate financial analysts to plan the upgrade. Below are visual representations of the analysts' predictions for yearly net profit after the upgrade. Which chart predicts a higher profit in 2021?

## Chart 1



Chart 2


1. Determine the units in each chart.

Chart 1 gives the profit in millions of dollars per year, or $\frac{\$ \text { (millions) }}{\text { year }}$.
Chart 2 gives two sets of units: thousands of heaters produced per heaters (thousands)
year and dollars of profit per heater, or $\frac{\text { year }}{}$ and
dollars year
heater
2. Identify the quantities of interest.

We are only interested in the profit prediction for the year 2021.
Chart 1 predicts a profit of $\$ 8$ million for that year, or $\frac{\$ 8,000,000}{1 \text { year }}$.
Chart 2 predicts 200,000 heaters will be produced in 2021, with
a profit of $\$ 50$ per heater. Symbolically, this is equivalent to
$\frac{200,000 \text { heaters }}{1 \text { year }}$ and $\frac{\$ 50}{1 \text { heater }}$.

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3. Identify the target units.

We want to identify which chart predicts the larger profit in 2021.
This corresponds to the unit ratio $\frac{\text { dollars }}{\text { year }}$.
4. Convert the profit predictions to the target units and compare.

Chart 1's prediction, $\frac{\$ 8,000,000}{1 \text { year }}$, is already in the desired units. To convert Chart 2's prediction to these units, multiply the number of heaters produced per year by the profit per heater.

| $\frac{200,000 \text { heaters }}{1 \text { year }} \cdot \frac{\$ 50}{1 \text { heater }}$ | Set up the conversion. |
| :--- | :--- |
| $\frac{200,000 \text { heaters }}{1 \text { year }} \cdot \frac{\$ 50}{1 \text { heater }}$ | Cancel units appearing in the <br> numerator and denominator. |
| $\frac{\$ 200,000 \bullet 50}{1 \text { year }}$ | Simplify. |
| $\frac{\$ 10,000,000}{1 \text { year }}$ | Multiply. |

Chart 2 predicts a profit of $\$ 10$ million in 2021 , which is greater than Chart 1's prediction of $\$ 8$ million.

