Guided Practice 2.1.1

Example 1

James earns \$15 per hour as a teller at a bank. In one week he pays 17% of his earnings in state and federal taxes. His take-home pay for the week is \$460.65. How many hours did James work?

1. Read the problem statement carefully.

2. Reread the scenario and make a list of the known quantities.

   James earns \$15 per hour.

   James pays 17% of his earnings in taxes.

   After taxes, his take-home pay for the week is \$460.65.

3. Read the statement again, identifying the unknown quantity or variable.

   The scenario asks for James's hours for the week. The variable to solve for is hours.

4. Create an equation from the known quantities and variable(s).

   James's pay for the week was \$460.65.

   $\underline{\qquad\qquad} = 460.65$

   He earned \$15 an hour. Let h represent hours.

   $15h$

   He paid 17% in taxes. Written as a decimal, 17% = 0.17. Since taxes are subtracted from pay, use a negative sign: –0.17. Multiply $15h$ by –0.17.

   $-0.17(15h)$

   Put this information all together.

   $15h - 0.17(15h) = 460.65$

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5. Solve the equation for the variable.

$15h - 0.17(15h) = 460.65$	Equation from the previous step
$15h - 2.55h = 460.65$	Multiply –0.17 and 15*h*.
$12.45h = 460.65$	Combine like terms 15*h* and –2.55*h*.
$\frac{12.45h}{12.45} = \frac{460.65}{12.45}$	Divide both sides by 12.45.
$h = 37$	Simplify.

6. Interpret the solution in the context of the problem and convert to the appropriate units if necessary.

   In the equation, *h* represents hours worked. The scenario asked for hours and the quantity given was in terms of hours. No unit conversions are necessary.

   James worked 37 hours.

Example 2

Brianna has saved \$600 to buy a new TV. If the TV she wants costs \$1,800 and she saves \$20 a week, how many years will it take her to buy the TV?

1. Read the problem statement carefully.

2. Reread the scenario and make a list of the known quantities.

   The TV costs \$1,800.

   Brianna has already saved \$600.

   Brianna saves \$20 per week.

3. Read the statement again, identifying the unknown quantity or variable.

   The scenario asks for the number of years. This is tricky because the quantity is given in terms of weeks. The variable to solve for first, then, is weeks.

4. Create an equation from the known quantities and variable(s).

   Brianna's goal is to save $1,800.

   _______ = 1800

   Brianna has saved $600 so far and has to save more to reach her goal.

   600 + _______ = 1800

   Brianna is saving $20 a week for some unknown number of weeks to reach her goal. Let x represent the number of weeks.

   $600 + 20x = 1800$

5. Solve the equation for the variable.

$$\begin{aligned} 600 + 20x &= 1800 \\ -600 \qquad &\;\; -600 \\ \hline 20x &= 1200 \\ \frac{20x}{20} &= \frac{1200}{20} \\ x &= 60 \end{aligned}$$

6. Interpret the solution in the context of the problem and convert to the appropriate units if necessary.

   In the equation, x represents weeks. The problem statement asks for the number of years it will take Brianna to save for the TV, so convert weeks to years. There are 52 weeks in a year.

$$\frac{1\text{ year}}{52\text{ weeks}}$$

$$60\text{ weeks} \bullet \frac{1\text{ year}}{52\text{ weeks}}$$

$$60\ \cancel{\text{weeks}} \bullet \frac{1\text{ year}}{52\ \cancel{\text{weeks}}} \approx 1.15\text{ years}$$

   Brianna will need approximately 1.15 years, or a little over a year, to save for her TV.

Example 3

Suppose two brothers who live 55 miles apart decide to have lunch together. To prevent either brother from driving the entire distance, they agree to leave their homes at the same time, drive toward each other, and meet somewhere along the route. The older brother drives cautiously at an average speed of 60 miles per hour. The younger brother drives faster, at an average speed of 70 mph, but still within the speed limit. How long will it take the brothers to meet each other?

1. Read the problem statement carefully.

2. Reread the scenario and list the known quantities.

   Problems involving "how fast," "how far," or "how long" can often be solved with the distance equation, $d = rt$, where d is distance, r is the rate of speed, and t is time.

   From the scenario, we know each brother's rate of speed, and the total distance they have to drive to meet each other. Make a list:

   The older brother's rate is 60 mph.

   The younger brother's rate is 70 mph.

   The sum of their distances is 55 miles.

3. Read the statement again, identifying the unknown quantity or variable.

   The scenario asks "how long" it will take, so the variable is time, t.

4. Create an equation from the known quantities and variable(s).

   Step 2 showed that the distance equation is $d = rt$ or $rt = d$. Together the brothers will travel a distance, d, of 55 miles.

   (older brother's rate)(t) + (younger brother's rate)(t) = 55

   The rate r of the older brother is 60 mph and the rate of the younger brother is 70 mph. Substitute these values into the expression.

   $60t + 70t = 55$

   Create a table to see this another way.

	Rate (r)	**Time (t)**	**Distance (d)**
Older brother	60 mph	t	$d = 60t$
Younger brother	70 mph	t	$d = 70t$

   Together, they traveled 55 miles, so add the distance equations based on each brother's rate.

   $60t + 70t = 55$

5. Solve the equation for the variable.

$60t + 70t = 55$	Equation from the previous step
$130t = 55$	Combine the like terms, $60t$ and $70t$.
$\frac{130t}{130} = \frac{55}{130}$	Divide both sides by 130.
$t \approx 0.423$	Simplify.

   Note: The answer was rounded to the nearest thousandth. Although we do not need this precision, it is good practice to retain more digits than are needed until the last step to avoid rounding errors. When talking about meeting someone, it is highly unlikely that anyone would report a time that is broken down into decimals, which is why the next step will convert the units.

6. Interpret the solution in the context of the problem and convert to the appropriate units if necessary.

   In the equation, t represents hours. Automobile speeds in the United States are typically given in miles per hour (mph). Therefore, this unit of measurement is appropriate. However, typically portions of an hour are reported in minutes unless the time given is $\frac{1}{2}$ of an hour. Convert 0.423 hours to minutes using 60 minutes = 1 hour.

   $$60 \text{ min} = 1 \text{ hr}$$

   $$0.423 \text{ hr} \bullet \frac{60 \text{ min}}{1 \text{ hr}}$$

   $$0.423 \cancel{\text{hr}} \bullet \frac{60 \text{ min}}{1 \cancel{\text{hr}}} \approx 25.38 \text{ minutes}$$

   Here again, rarely would a person report that they are meeting someone in 25.38 minutes. In this case, there is a choice of rounding to either 25 or 26 minutes. Either answer makes sense.

   The two brothers will meet each other in 25 or 26 minutes. ✓

Example 4

Think about the following scenarios. In what units should they be reported? Explain the reasoning.

- The rate at which water fills up a swimming pool
- The cost of tiling a kitchen floor
- The average speed of a falling object
- The rate at which a snail travels across a sidewalk
- The rate at which a room is painted

1. The rate at which water fills up a swimming pool

   A swimming pool, depending on the size, has between several gallons and hundreds of thousands of gallons of water.

   Think about water flowing out of a faucet and picture filling up a milk jug. How long does it take? Less than a minute? The point is that gallons of water can be filled in minutes.

   A small swimming pool could be reported in terms of gallons per minute, and a large swimming pool could be reported in terms of gallons per hour.

2. The cost of tiling a kitchen floor

   Think about how big rooms are. They can be small or rather large, but typically they are measured in feet. When calculating the area, the measurement units are square feet.

   Tiles cost in the dollar range, and are priced by the square foot.

   Report the cost of tiling a kitchen floor in dollars per square foot.

3. The average speed of a falling object

   Think about how fast an object falls when you drop it from shoulder-height. How far is it traveling from your shoulder to the ground? It travels several feet (or meters).

   How long does it take before the object hits the ground? It only takes a few seconds.

   Report the average speed of a falling object in terms of feet or meters per second.

4. The rate at which a snail travels across a sidewalk

   The context of the problem will determine the correct units. Think about how slowly a snail moves. Would a snail be able to travel at least one mile in an hour? Perhaps it makes more sense to report the distance in a smaller unit. Report the snail traveling across the sidewalk in feet per minute.

   If comparing speeds of other animals to the snail's rate, and the animals' rates are being reported in miles per hour, then it makes sense to report the snail's rate in miles per hour, too.

5. The rate at which a room is painted

   Think about how long it takes to paint a room. It takes longer than several minutes. It would probably take hours.

   How is the surface area of a wall typically measured? It's usually measured in square feet.

   Report the painting of a room in square feet per hour.

Example 5

Ernesto built a wooden car for a soap box derby. He is painting the top of the car blue and the sides black. He already has enough black paint, but needs to buy blue paint. He needs to know the approximate area of the top of the car to determine the size of the container of blue paint he should buy. He measured the length to be 9 feet $11\frac{1}{4}$ inches, and the width to be $\frac{1}{2}$ inch less than 3 feet. What is the surface area of the top of the car? What is the most accurate area measurement Ernesto can use to buy his paint?

1. Read the problem statement carefully.

2. Reread the scenario and make a list of the known quantities.

   The length is 9 feet 11.25 inches.

   The width is 35.5 inches (3 feet = 36 inches; $36 - \frac{1}{2}$ inch = 35.5 inches).

3. Read the statement again, identifying the unknown quantity or variable.

   The scenario asks for the surface area of the car's top.

   Work with the accuracy component after calculating the surface area.

4. Create an equation from the known quantities and variable(s).

   The surface area will require some assumptions. A soap box derby car is tapered, meaning it is wider at one end than it is at another. To be sure Ernesto has enough paint, he assumes the car is rectangular with the width being measured at the widest location. Use the formula for area:

   $A = \text{length} \bullet \text{width} = lw$

   For step 2, we listed the length and width, but they are not in the same units. The length is given in feet and inches; the width is given in inches.

   Convert the length, 9 feet 11.25 inches, to inches.

   $9(12) + 11.25 = 119.25$ inches

   The width is 35.5 inches.

   Substitute the length and width into the formula $A = lw$.

   $A = lw$

   $A = (119.25) \bullet (35.5)$

5. Solve the equation for the variable.

   $A = 119.25 \bullet 35.5$

   $A = 4233.375$

   This gives a numerical result for the surface area, but the problem asks for the most accurate surface area measurement that can be calculated based on Ernesto's initial measurements. It is good practice to retain all three digits after the decimal place until the problem is complete to avoid inaccuracies introduced by rounding.

6. Interpret the solution in the context of the problem and convert to the appropriate units if necessary.

   In the equation, A represents area in square inches. When buying paint, the hardware store associate will ask how many square feet need to be covered. Ernesto has his answer in terms of square inches. Convert to square feet.

   There are 144 square inches in a square foot.

   $$1\text{ ft}^2 = 144\text{ in}^2$$

   $$4233.375\text{ in}^2 \bullet \frac{1\text{ ft}^2}{144\text{ in}^2}$$

   $$4233.375\ \cancel{\text{in}^2} \bullet \frac{1\text{ ft}^2}{144\ \cancel{\text{in}^2}} = 29.3984375\text{ ft}^2$$

7. Round the result to the appropriate number of places.

   Ernesto's initial measurement with the fewest digits after the decimal is 35.5, which is reported to the nearest tenth. Thus, the final answer should also be rounded to the nearest tenth.

   $$29.3984375\text{ ft}^2 \approx 29.4\text{ ft}^2$$

   The surface area of the top of Ernesto's car is approximately 29.4 ft^2. This is the most accurate area measurement that Ernesto can use to buy his paint.