Prerequisite Skills

This lesson requires the use of the following skills:

- reading and interpreting data from charts and tables (6.EE.9)
- understanding slope (8.EE.5)

Introduction

In previous lessons, we found the slope of the graph of a linear function using the slope formula, $m=\frac{y_2-y_1}{x_2-x_1}$. We also identified the slope of a line from a given equation by rewriting the equation in **slope-intercept form**, $y = mx + b$, where m is the slope of the line and b is the y-intercept. By calculating the slope, we are able to determine the **rate of change**, or the ratio between the change in the y-values and the corresponding change in the x-values. The rate of change can be determined from graphs, tables, and equations.

Key Concepts

- A **ratio** is the relation between two quantities. A ratio can be expressed in words, or as a fraction, decimal, or percent.
- A rate of change is a ratio describing how one quantity changes with respect to another quantity.
- Slope is a way to describe the rate of change of a function.
- The **slope** of a line is the ratio of the change in y-values and the change in x-values between two points.
- A positive rate of change signifies that a function increases as x increases.
- A negative rate of change signifies that a function decreases as x increases.
- Linear functions have a constant rate of change, meaning that the y-value increases or decreases the same amount for a given change in x. Linear functions may also be constant, which means that they don't increase or decrease.
- The rate of change of a function within an interval, or a continuous portion of a function, can be calculated.

- The rate of change of a function within an interval is the average rate of change for the interval, defined by values of x in the domain of a function.
- Closed intervals can be noted using the format $[a, b]$, where a represents the initial x-value of the interval and b represents the final x-value of the interval. This is a closed interval because it includes the endpoints of the interval. Another way to state that the interval is closed is $a \le x \le b$.
- A linear function that has a slope of 0 is a horizontal line.
- Vertical lines have an **undefined slope**. This occurs when the denominator of the slope formula is equal to 0.

Calculating Rate of Change from a Table
1. Choose two points from the table. 2. Assign the coordinates of one point to be (x_1, y_1) and the other point to be (x_2, y_2). 3. Substitute the values into the slope formula, $m=\frac{y_2-y_1}{x_2-x_1}$. 4. If the difference between the x-coordinates is not 0, the result is the rate of change between the two points.

- The rate of change between any two points of a linear function will be equal.

Calculating Rate of Change from an Equation of a Linear Function
1. Transform the given linear equation into slope-intercept form, $f(x) = mx + b$. 2. Identify the slope of the line as m from the equation. 3. The slope of the linear function is the rate of change between all pairs of points on the graph of the function.

- To determine the rate of change of a function from a graph, first identify the coordinates of the endpoints of the interval being observed.
- Sometimes it is necessary to estimate the values for y.
- The resulting calculation may be an estimation of the rate of change for the interval identified for the given function.

Estimating Rate of Change from a Graph
4. Determine the interval to be observed. 5. Identify (x_1, y_1) as the initial point of the interval. 6. Identify (x_2, y_2) as the endpoint of the interval. 7. Substitute (x_1, y_1) and (x_2, y_2) into the slope formula to calculate the rate of change. 8. The result is the estimated rate of change for the interval between the two points identified.

Common Errors/Misconceptions

- incorrectly choosing the values of the indicated interval to calculate the rate of change
- substituting incorrect values into the slope formula
- interpreting interval notation as coordinates