Prerequisite Skills

This lesson requires the use of the following skills:

- recognizing patterns (3.OA.9)
- understanding the properties of a function (8.F.4)
- understanding function notation (F–IF.2)

Introduction

A **sequence** is an ordered list of numbers. The numbers, or terms, in the ordered list can be determined by a formula that is a function of the position of each term in the list. So if we have a sequence A determined by a function f, the terms of the sequence will be:

$$A = a_1, a_2, a_3, \ldots, a_n, \text{ where } a_1 = f(1), a_2 = f(2), a_3 = f(3), \ldots, a_n = f(n)$$

Unlike a typical function on a variable, there are no fractional elements in the domain of a sequence. Each value of n is a whole number. Just as no one can place 1.25th in a race, there is no 1.25th term in a sequence. Therefore, the domain of f includes elements from the set {1, 2, 3, ...}. These numbers are a subset of the set of integers, and are called the **natural numbers**. Natural numbers are the numbers we use for counting. Every element of the domain of a sequence is individually separate and distinct, so we say that a sequence is a **discrete** function.

Key Concepts

- Sequences are ordered lists that can be expressed as functions.
- The domain of a function that generates a sequence is the set of all natural numbers.
- There are two ways sequences are generally defined—recursively and explicitly.
- An **explicit formula** is a formula used to find the nth term of a sequence. A sequence is defined explicitly (that is, with an explicit formula) if a function is given.
- A **recursive formula** is a formula used to find the next term of a sequence when the previous term or terms are known. In a recursive sequence (that is, one with a recursive formula), the next term is based on the preceding term and the commonality between terms.
- For this lesson, most sequences have a common difference.

- Sequences that have a common difference between consecutive terms are called **arithmetic sequences**.
- Some sequences, such as 4, 7, 12, 19, 28, ..., do not have a common difference.
- A **common difference** is a number that is added to each consecutive term in an arithmetic sequence. To determine the common difference in an arithmetic sequence, subtract the first term from the second term. Then subtract the second term from the third term, and so on.
- If a common difference exists, $a_n - a_{n-1}$ is a constant. So, $a_4 - a_3 = a_3 - a_2 = a_2 - a_1$.

Explicit Formulas

- Explicitly defined sequences provide a formula, or function, that will generate each term. For example, $a_n = 2n + 3$. Explicit formulas do not rely on knowing a previous term.
- To find a_n, simply substitute the value of n (representing the nth term) into the formula.

Recursive Formulas

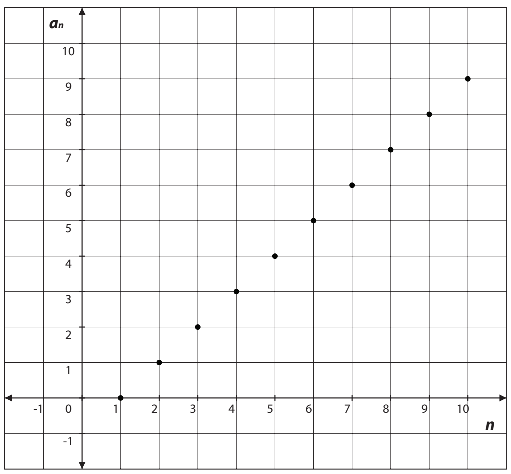
- A second way to define a sequence is with a recursive formula. In a sequence that is defined recursively, each term is a function of the term or terms that come before it. For example, $a_n = a_{n-1} + 2$, where n is the number of the term and a_n is the value of that term.

Instruction

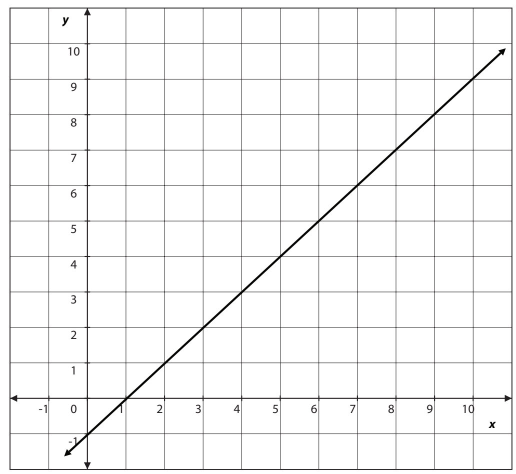
Graphing Sequences

- A sequence can be graphed. Its domain is the set of natural numbers. Compare the following graphs. The first graph represents the sequence $a_n = n - 1$, and the second graph represents the line $f(x) = x - 1$.

Graph of a sequence



Graph of a linear function



- Notice that the sequence only has values for $n = 1, 2, 3, 4$, etc., and consists of discrete points.

Common Errors/Misconceptions

- not realizing that a sequence can be written as a function
- not understanding that the graph of a sequence consists of a set of discrete points