Georgia Standards of Excellence

Algebra I

$f(x)$ $\angle ABC$ Q_1

Student Workbook

Unit 3

WALCH®
HIGH SCHOOL MATH

1 2 3 4 5 6 7 8 9 10
ISBN 978-0-8251-8504-5

J. Weston Walch, Publisher
Portland, ME 04103
www.walch.com
Printed in the United States of America

WALCH®
HIGH SCHOOL MATH

Table of Contents

Introduction

The *Georgia Standards of Excellence Algebra I Student Workbook* includes all of the student pages from the Teacher Resource necessary for day-to-day classroom instruction. This includes:

- Warm-Ups
- Problem-Based Tasks
- Practice Problems
- Station Activity Worksheets

In addition, it provides Scaffolded Guided Practice examples that parallel the examples in the TRB. This supports:

- Students taking notes during class
- Students working problems for preview or additional practice
- Teachers using the TRB to review Guided Practice

The workbook includes the first Guided Practice example with step-by-step prompts for solving, and the remaining Guided Practice examples without prompts, available for various instruction and practice options. Sections for taking notes are provided at the end of each sub-lesson. Additionally, blank coordinate planes are included at the end of the full lesson, should graphing be required. And directly following this introduction, useful formulas are provided for student reference.

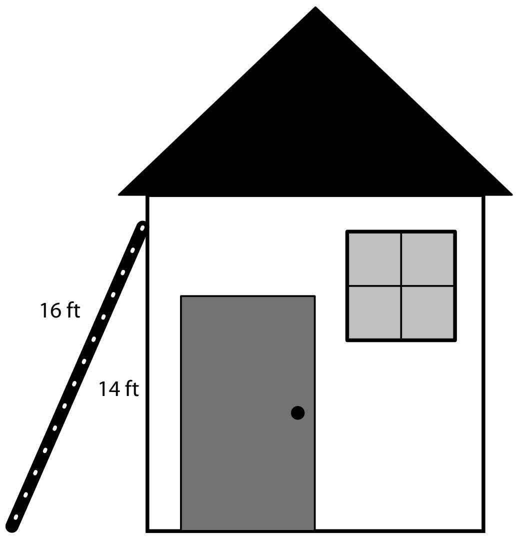
The workbook is printed on perforated paper to facilitate submission of assignments and three-hole punched to allow for storage in a binder.

Student Workbooks with Scaffolded Practice save time for teachers as well as copying expenses, ensure that students have the materials they need, and provide an additional, flexible instructional resource.

Lesson 3.1.1: Taking the Square Root of Both Sides

Warm-Up 3.1.1

A 16-foot ladder leans against the side of a house. The ladder reaches 14 feet up the side of the house.



1. How far is the base of the ladder from the house?

2. Suppose the ladder is moved 2 feet closer to the house. Now how far up the side of the house does the ladder reach?

Scaffolded Practice 3.1.1

Example 1

Solve $2x^2 - 5 = 195$ for x.

1. Isolate x^2.

2. Use a square root to find all possible solutions to the equation.

continued

Example 2

Solve $4(x + 3)^2 - 10 = -6$ for x.

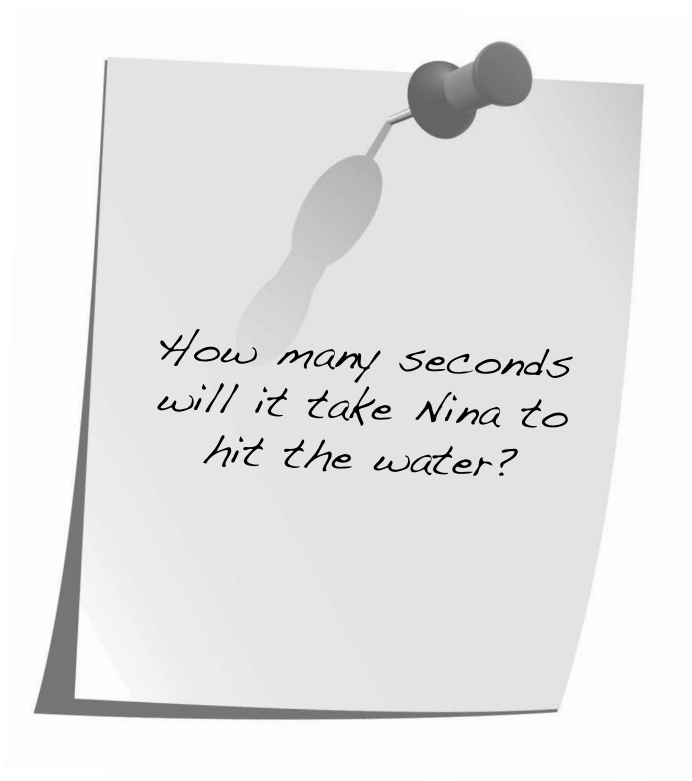
Example 3

Solve $(x - 1)^2 + 15 = -1$ for x.

Problem-Based Task 3.1.1: Time to Splash

SMP	
1 ✓	2
3	4 ✓
5	6
7 ✓	8

Nina dives into a pool from a platform 3.75 feet above the water. Her height above the water in feet, x seconds after she jumps, is given by the expression $-5(x - 0.5)^2 + 5$. How many seconds will it take Nina to hit the water?



Practice 3.1.1: Taking the Square Root of Both Sides A

Solve each equation for x.

1. $x^2 = 81$

2. $x^2 = -25$

3. $x^2 - 5 = 4$

4. $(x + 3)^2 = 1$

5. $(x + 3)^2 + 7 = -2$

6. $4(x - 10)^2 = 25$

Use what you know about square roots to complete problems 7–10. Round to the nearest hundredth, if necessary.

7. When does a quadratic equation in the form $ax^2 + b = c$ have two real, rational solutions?

8. The area of a square with sides of length s is given by s^2. The area of a square is 40 square centimeters. What is the length of one side of the square, rounded to the nearest hundredth?

9. The area of a circle with radius r is given by πr^2. The area of a circle is 60 square millimeters. What is the radius of the circle, rounded to the nearest hundredth?

10. The surface area of a cube with edges of length a is given by $6a^2$. If the surface area of a cube is 200 square inches, what is the length of each edge of the cube, rounded to the nearest hundredth?

Practice 3.1.1: Taking the Square Root of Both Sides

B

Solve each equation for x.

1. $x^2 = 4$

2. $x^2 + 8 = 4$

3. $x^2 + 5 = -3$

4. $(x - 4)^2 = 40$

5. $2(x + 6)^2 - 6 = -6$

6. $8(x - 5)^2 = 56$

Use what you know about square roots to complete problems 7–10. Round to the nearest hundredth, if necessary.

7. When does a quadratic equation in the form $ax^2 + b = c$ have only one real solution?

8. The area of a square with sides of length s is given by s^2. The area of a square is 49 square inches. What is the length of one side of the square?

9. The area of a circle with radius r is given by πr^2. The area of a circle is 20π square units. What is the radius of the circle?

10. The surface area of a sphere with radius r is given by $4\pi r^2$. If the surface area of a sphere is 20 square feet, what is the radius of the sphere?

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Lesson 3.1.2: Factoring Expressions by the Greatest Common Factor

Warm-Up 3.1.2

Several graphic artists competing on a reality show must design a new logo for a struggling business. The artists are divided into three groups: Group A, Group B, and Group C. Each group must fit the logo within a rectangular space. To make the task more challenging, each group is assigned specific measurements for one or two of the dimensions of their rectangle—either just the area measurement, the area and the length, or the area and the width. The other dimensions of the rectangle can vary, as long as the assigned measurements are correct. Given the formula for the area of a rectangle, $A = lw$, and the requirement that all dimensions must be in whole numbers, use the information given in each problem to determine the requested information about the rectangles.

1. Group A is assigned an area of 30 square inches. How many different rectangles can be created, and what are the lengths and widths for each rectangle?

2. Group B is assigned a length of 6 inches and an area of $30x$ square inches. What is the width of this rectangle?

3. Group C is assigned a width of $3x$ and an area of $30x^3y$ square inches. What is the length of this rectangle?

Scaffolded Practice 3.1.2

Example 1

Factor the polynomial $12x^3 + 30x^2 + 42x$ by finding the greatest common factor (GCF). Verify your results using the Distributive Property.

1. Determine the common factors of the terms' coefficients and variables.

2. Write expressions for the remaining non-common factors.

3. Factor the original polynomial by writing it as the product of the GCF and a polynomial whose terms are the expressions found in the previous step.

4. Verify the result using the Distributive Property.

continued

Example 2

Factor the polynomial $35xy^4 - 14x^4y^2z + 56x^3y^3z^3$ by finding the GCF. Verify your results using the Distributive Property.

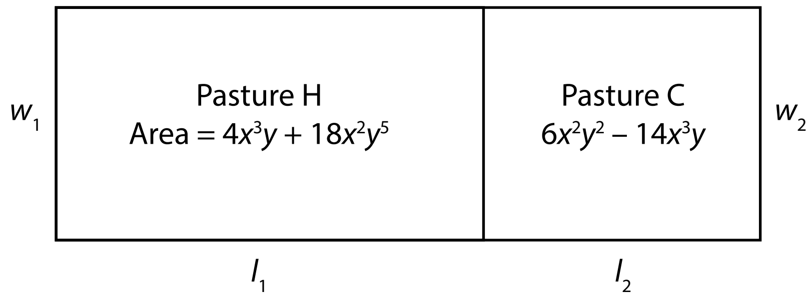
Example 3

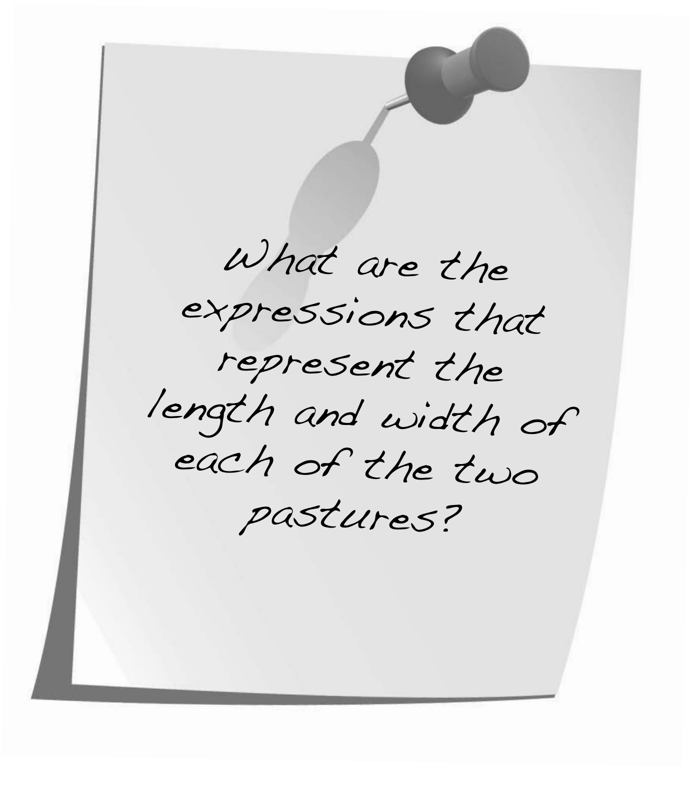
The polynomial $15x^2 - 3x$ represents the area of a rectangular garden plot in square yards, where the length of the garden is equal to the GCF. Determine expressions for the length and width of the garden.

Problem-Based Task 3.1.2: Out to Pasture

SMP
1 ✓ 2 ✓
3 4
5 6
7 ✓ 8

A farmer has a long rectangular field, which is divided into two smaller rectangular fields by a fence. The field on the left, Pasture H, is reserved for horses and has an area in square meters represented by the polynomial $4x^3y + 18x^2y^5$. The field's width is equal to the greatest common factor of the polynomial. The field on the right, Pasture C, is reserved for cows. Its area in square meters is $6x^2y^2 - 14x^3y$. What are the expressions that represent the length and width of each of the two pastures?





Practice 3.1.2: Factoring Expressions by the Greatest Common Factor A

For problems 1–6, factor each expression by the greatest common factor if a common factor exists, or state that the terms have no common factor.

1. $3x^3 + 5x^2$
2. $2x^2y - 8xy^2$
3. $y^4 + 2y^2$
4. $x^2 - 9x + 3$
5. $x^3y^2 - 2x^2y^3 + 5xy^2$
6. $7x - 21x^2y$

Use what you have learned about factoring polynomials by the GCF to complete problems 7 and 8.

7. Christopher has two bags of marbles. The number of marbles in the first bag can be represented by the monomial $45x^2y$, and the number of marbles in the other bag can be represented by $60x^3y^2$. What is the GCF of these two monomials?

8. An equilateral triangle has a perimeter of $(15x^3 + 33y^2)$ feet. What is the length of each side?

Use the following information to complete problems 9 and 10.

Samuel and Ariana are competing in a speed round for an open position on the math team. To win the spot, each student must factor the same polynomial expression, $12xyz^2 + 16x^2y^2z - 32x^2yz$, by finding the GCF.

9. Samuel's final result was $2xyz(6z + 8xy - 16x)$. Explain his error, if any.

10. Ariana's final result was $4xyz^2(3x + 4xy - 8xyz)$. Explain her error, if any.

Practice 3.1.2: Factoring Expressions by the Greatest Common Factor B

For problems 1–6, factor each expression by the greatest common factor if a common factor exists, or state that the terms have no common factor.

1. $s^3t^2 - 3s^2 + 2st^2$
2. $4x^3y + 2x^5y^3$
3. $5 + x - 2x^2$
4. $x^4y + x^3y^2 + x^2y^3 + xy^4$
5. $x^2 + 3xy + y^2$
6. $x^4 + x^{14}$

Use what you have learned about factoring polynomials by the GCF to complete problems 7–10.

7. A square has a perimeter of $(52x^3y - 24z^2)$ inches. What is the length of each side?
8. The area of a rectangular swimming pool can be represented by the expression $(3x^4 - 12x^3 + 15x^2)$ square feet. The length of the pool is equal to the GCF. What are the length and width of the pool?
9. Explain why the polynomial $17x^3 + 24y^2$ cannot be factored by the GCF.
10. A regular pentagon (with all sides equal) has a perimeter of $(15a^2b^3 - 20c^2 + 60)$ meters. What is the length of each side?

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Lesson 3.1.3: Factoring Expressions with *a* = 1

Warm-Up 3.1.3

Erica and her dad want to plant a rectangular vegetable garden in their backyard, and are trying to determine the best size. They need to know how much area will be taken up by the different options they have in mind for the length and width. Use the formula for the area of a rectangle, $A = lw$, to complete the following problems.

1. If the length is $(x + 5)$ feet and the width is $(x + 3)$ feet, what expression represents the area of the garden?

2. If the length is $(x + 4)$ feet and the width is $(x - 4)$ feet, what expression represents the area of the garden?

3. If they want the area to be $(x^2 + 7x)$ square feet with a length of x feet, what does the width need to be?

Scaffolded Practice 3.1.3

Example 1

Factor $9x^2 - 16$, and then verify your results.

1. Determine any common factors of the given binomial, if common factors exist.

2. Determine if the given binomial meets the conditions of the difference of two squares.

3. Use the pattern $x^2 - y^2 = (x + y)(x - y)$ to factor the given binomial.

4. Multiply the factors to verify that they result in the original binomial.

continued

Example 2

Factor $5y^2 - 45$, and then verify your results.

Example 3

The polynomial expression $x^2 + 7x - 8$ represents the area in square feet of the Bingham family's rectangular backyard. Factor this polynomial to find the expressions that represent the length and the width of the backyard, and then verify your results.

Example 4

Factor $2a^2 - 16a + 32$, and then verify your results.

Example 5

Factor $x^2 - 18x + 81$, and then verify your results.

Problem-Based Task 3.1.3: Don't Drop the Ball!

A city is installing two identical rectangular volleyball courts at the local park. The expression $2x^2 + 6x - 36$ represents the combined area of both courts. If each court is 9 meters by 18 meters, what does x equal?

SMP
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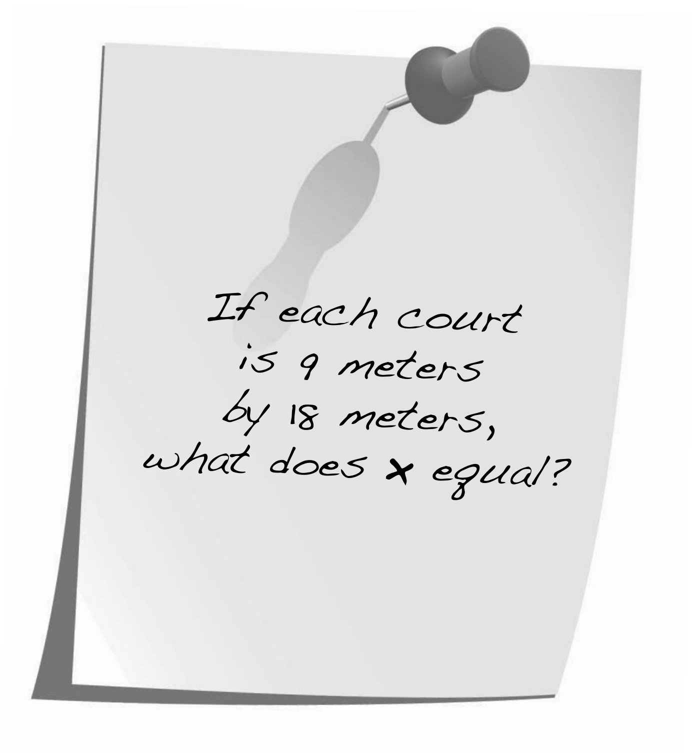
3 4
5 6
7 ✓ 8



Practice 3.1.3: Factoring Expressions with *a* = 1 A

For problems 1–7, factor each expression as much as possible. If the expression cannot be factored, write "not factorable."

1. $y^2 - 100$

2. $x^2 - 9x + 14$

3. $x^2 + 16x + 64$

4. $b^2 + 4$

5. $7a^2 - 28$

6. $4x^2 + 32x - 36$

7. $6x^2 - 54y^2$

For problems 8–10, each expression represents the area of a rectangle or square. Factor each expression to find the expressions that represent the length and width of each figure.

8. $(a^2 - 14a + 49)$ square feet

9. $(4x^2 - 25)$ square meters

10. $(y^2 + 3y - 10)$ square inches

Practice 3.1.3: Factoring Expressions with *a* = 1 B

For problems 1–7, factor each expression as much as possible. If the expression cannot be factored, write "not factorable."

1. $n^2 - 36$

2. $x^2 - 9x + 20$

3. $y^2 - 20y + 100$

4. $x^2 + 9$

5. $8b^2 - 8$

6. $5x^2 + 10x - 40$

7. $10n^2 - 90m^2$

For problems 8–10, each expression represents the area of a rectangle or square. Factor each expression to find the expressions that represent the length and width of each figure.

8. $(n^2 - 24n + 144)$ square feet

9. $(9y^2 - 100)$ square meters

10. $(x^2 + 7x - 18)$ square inches

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Lesson 3.1.4: Factoring Expressions with $a > 1$

Warm-Up 3.1.4

Rosalind is painting three rectangular pictures. The first, a portrait, will have an area of $(x^2 + 5x + 4)$ square inches. Next, she will paint a landscape with an area of $(x^2 + 10x + 24)$ square inches. Finally, she plans to create a modern art piece with an area of $(x^2 - 16)$ square inches. Use the formula for the area of a rectangle, $A = lw$, to complete the following problems.

1. If the length of the portrait is $(x + 4)$ inches, what is the portrait's width?

2. What expressions represent the length and width of the landscape painting, given that the length is longer than the width?

3. For the modern art piece, how much larger is the width than the length?

Scaffolded Practice 3.1.4

Example 1

Factor $5y^2 - 6y + 10y - 12$ by grouping, and then verify your results.

1. Group the terms of the given polynomial according to their common factors, if common factors exist.

2. Factor out common factors from each set of grouped terms, and rewrite the expression from the previous step.

3. Use the Distributive Property to rewrite the expression as two factors.

4. Distribute the result of the previous step to confirm that the original polynomial was factored correctly.

continued

Example 2

Factor $6x^2 - 7x - 5$, and then verify your results.

Example 3

Factor $36x^2 - 54x + 8$, and then verify your results.

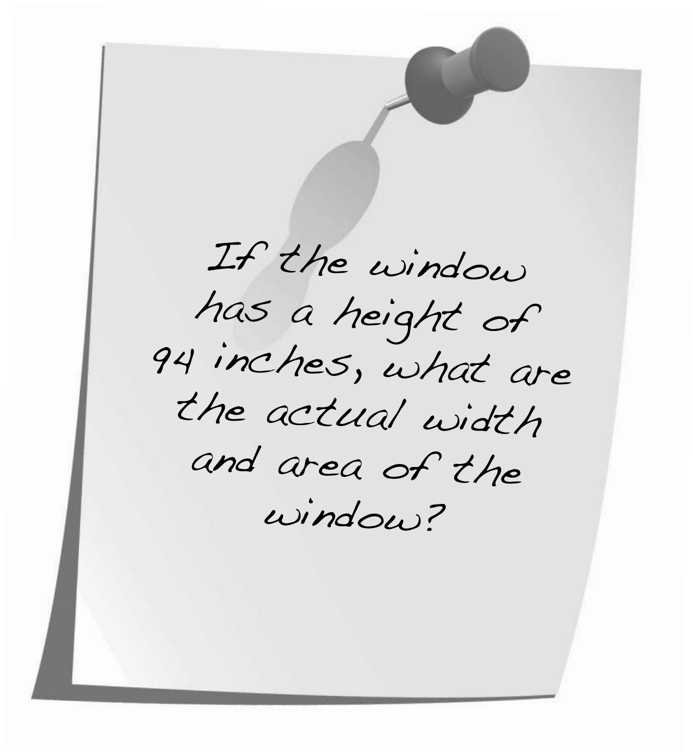
Example 4

The polynomial expression $10a^2 + 87a - 27$ represents the area in square yards of a new rectangular playground at the town park. Factor the polynomial to determine the expressions that represent the length and the width of the playground. Verify your results.

Problem-Based Task 3.1.4: Window Installation

SMP
1 ✓ 2 ✓
3 4 ✓
5 6
7 ✓ 8

A construction company is installing a rectangular pane of stained glass at a local museum. The expression $54x^2 + 69x + 20$ represents the area of the window in square inches, and the expression $6x + 5$ represents the width of the window in inches. If the window has a height of 94 inches, what are the actual width and area of the window? Recall that the area of a rectangle is equal to its width times its length (in this case, the height of the window).



Practice 3.1.4: Factoring Expressions with $a > 1$ A

For problems 1–3, factor each expression by grouping.

1. $15x^2 - 5x + 21x - 7$

2. $3n^2 + 2n + 24n + 16$

3. $4y^2 - 18y - 14y + 63$

For problems 4 –7, factor each trinomial.

4. $3x^2 + 11x - 20$

5. $3a^2 + 39a + 108$

6. $25n^2 + 20n + 4$

7. $12y^2 - 46y + 14$

For problems 8–10, each expression represents the area of a rectangle or square. Factor each polynomial to find the expressions that represent the length and width of each figure.

8. $(9x^2 + 42x + 49)$ square inches

9. $(5x^2 + 14x - 3)$ square feet

10. $(6x^2 + 17x - 14)$ square meters

Practice 3.1.4: Factoring Expressions with $a > 1$ B

For problems 1–3, factor each expression by grouping.

1. $8x^2 - 14x + 12x - 21$

2. $8a^2 + 16a - 3a - 6$

3. $10y^2 - 2y + 25y - 5$

For problems 4–7, factor each trinomial.

4. $2x^2 - 21x + 49$

5. $16a^2 - 24a + 9$

6. $5x^2 + 45x + 90$

7. $30y^2 + 34y - 8$

For problems 8–10, each expression represents the area of a rectangle or square. Factor each polynomial to find the expressions that represent the length and width of each figure.

8. $(6x^2 + 23x + 10)$ square inches

9. $(3x^2 + 22x - 16)$ square feet

10. $(10x^2 - 21x + 9)$ square meters

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Lesson 3.1.5: Solving Quadratic Equations by Factoring

Warm-Up 3.1.5

The Jacksons just had an in-ground pool built in their backyard. The pool is 5 feet deep and 12 feet wide. The expression $x + 5$ represents the pool's length and the expression $5x^2 - 15x - 200$ represents its volume. Use the fact that the volume equals the length times the width times the height (or depth) to answer the following questions.

1. What expression represents the width of the pool?

2. What is the value of x?

3. What is the actual length of the pool in feet?

4. What is the actual volume of the pool in cubic feet?

Scaffolded Practice 3.1.5

Example 1

Solve $x^2 + 3x = 0$ for x.

1. Write the equation in standard form, $ax^2 + bx + c = 0$, and determine values for a, b, and c.

2. Factor the polynomial and rewrite the equation.

3. Set each factor equal to 0 and solve for x.

4. Check the solutions by substituting them into the original equation.

continued

Example 2

Solve $4x^2 - 26x = 14$ for x.

Example 3

The area of the Baker family's rectangular driveway is 324 square feet. The length is 3 feet larger than twice the width. What are the length and width of the driveway?

Problem-Based Task 3.1.5: Lost Sunglasses

SMP	
1 ✓	2 ✓
3	4 ✓
5	6
7 ✓	8

Alyson lost her sunglasses while riding a roller coaster. The coaster was 144 feet high and traveling uphill at a rate of 40 feet per second when the sunglasses fell. The equation for the sunglasses' height (h) above the ground after t seconds is $h = -16t^2 + 40t + 144$. How long did it take Alyson's sunglasses to reach the ground?

Practice 3.1.5: Solving Quadratic Equations by Factoring A

For problems 1–7, solve each quadratic equation by factoring.

1. $x^2 - 2x - 48 = 0$

2. $2y^2 + 9y = 35$

3. $5n^2 - 9n = 0$

4. $2x^2 - 32 = 0$

5. $3y^2 - 24y = -45$

6. $60a^2 - 190a = 70$

7. $(x + 4)(x - 8) = 28$

For problems 8–10, each given equation represents the height (h) of an object above the ground after it has traveled in the air for t seconds. Solve each problem using the provided information.

8. A child throws a water balloon down out of a window. Substitute 0 for h into the equation $h = -16t^2 - 10t + 6$ to determine how many seconds it takes for the water balloon to reach the ground.

9. A person tosses a coin down from a balcony into a fountain below. Substitute 12 for h into the equation $h = -5t^2 - 2t + 36$ to determine how many seconds it will take before the coin passes a sign that is 12 feet above the ground.

10. A boater launches a firework up into the air. Substitute 125 for h into the equation $h = -5t^2 + 50t$ to determine how many seconds it will take before the firework reaches its maximum height of 125 meters and explodes.

Practice 3.1.5: Solving Quadratic Equations by Factoring B

For problems 1–7, solve each quadratic equation by factoring.

1. $x^2 - 13x + 36 = 0$

2. $3y^2 + 13y = -14$

3. $2b^2 - 11b = 0$

4. $3x^2 - 12 = 0$

5. $5y^2 - 35y = 40$

6. $12n^2 + 22n = -8$

7. $(x - 6)(x - 5) = 6$

For problems 8–10, each given equation represents the height (h) of an object or animal above the ground after it has traveled in the air for t seconds. Solve each problem using the provided information.

8. A bird swoops down out of a tree to the ground below. Substitute 0 for h into the equation $h = -16t^2 - 12t + 40$ to find how many seconds it takes for the bird to reach the ground.

9. A squirrel knocks an acorn off a roof. Substitute 14 for h into the equation $h = -5t^2 - 3t + 40$ to find how many seconds it will take before the acorn passes a window that is 14 feet above the ground.

10. A hobbyist launches a model rocket up into the air. Substitute 80 for h into the equation $h = -5t^2 + 40t$ to find out how many seconds it will take before the rocket reaches its maximum height of 80 meters.

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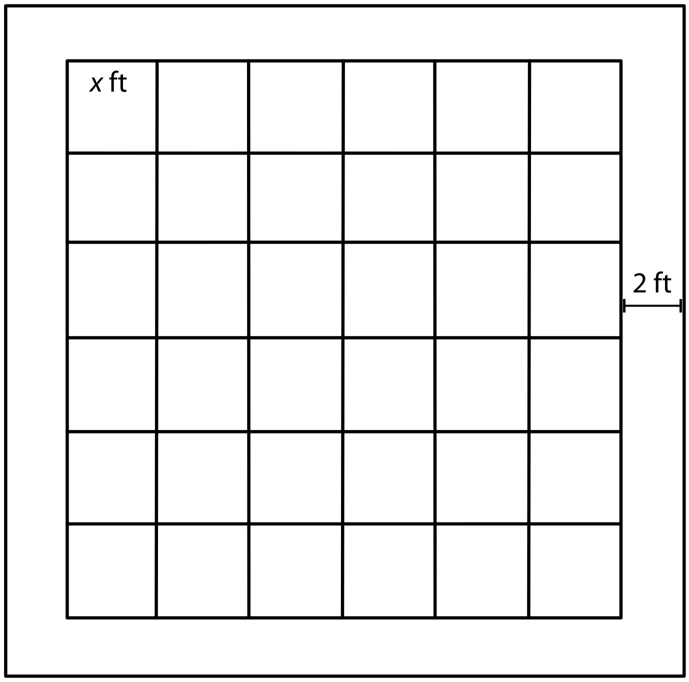
Lesson 3.1.6: Completing the Square

Warm-Up 3.1.6

Pavers are factory-made concrete tiles used in landscaping. Each side of a square patio is 6 pavers long. The length of one side of the patio can be represented by the term $6x$, where x is the length of one paver in feet.



1. Write an expression for the area of the patio covered by pavers.

2. A 2-foot gravel border surrounds the patio. Write an expression for the total area of the patio including the gravel border.

Scaffolded Practice 3.1.6

Example 1

Solve $x^2 - 8x + 16 = 4$.

1. Determine if $x^2 - 8x + 16$ is a perfect square trinomial.

2. Write the left side of the equation as a binomial squared.

3. Take the square root of both sides of the equation to solve for x.

4. Determine the solution(s).

continued

Example 2

Solve $x^2 + 6x + 4 = 0$ by completing the square.

Example 3

Solve $5x^2 - 50x - 120 = 0$ by completing the square.

Problem-Based Task 3.1.6: Curve Ball

SMP	
1	2 ✓
3	4 ✓
5	6
7 ✓	8

The height of a baseball in feet after it is thrown is represented by $-16x^2 + 32x + 5$, where x is the time in seconds. After how many seconds will the ball be at a height of 7 feet?

Practice 3.1.6: Completing the Square

A

For problems 1–4, find the value of *c* so that the expression is a perfect square trinomial.

1. $x^2 + 18x + c$

2. $x^2 - 24x + c$

3. $x^2 + 15x + c$

4. $x^2 + x + c$

Solve problems 5–7 by completing the square.

5. $x^2 + 10x = 0$

6. $x^2 + 12x - 13 = 0$

7. $3x^2 + 2x - 7 = 0$

Use what you know about completing the square to solve problems 8–10. Determine whether your answers are reasonable and explain why or why not.

8. A rectangular porch has an area of 75 square feet. The length of the porch is 4 feet longer than the width. What is the width of the porch?

9. A pet owner throws a tennis ball for his dog to chase. The tennis ball's height in feet after it is thrown upward is given by $-16x^2 + 32x + 4$, where x represents the time in seconds after the ball was thrown. After how many seconds will the ball hit the ground?

10. The fuel economy in miles per gallon of a certain vehicle is given by $-0.01x^2 + 1.2x - 5.8$, where x is the car's speed in miles per hour. For what speed(s) does the car have a fuel economy of 22 miles per gallon?

Practice 3.1.6: Completing the Square

B

For problems 1–4, find c so that the expression is a perfect square trinomial.

1. $x^2 + 22x + c$

2. $x^2 + 100x + c$

3. $x^2 - 9x + c$

4. $x^2 - \frac{4}{5}x + c$

Solve problems 5–7 by completing the square.

5. $x^2 - 8x + 2 = 0$

6. $2x^2 + 2x = 5$

7. $x^2 + 4x = 21$

Use what you know about squares and factoring to solve problems 8–10. Determine whether your answers are reasonable and explain why or why not.

8. A dog pen has an area of 60 square feet. The width of the pen is 2 feet shorter than its length. Find the length of the pen.

9. A student kicks a ball during gym class. The ball's height in feet x seconds after being kicked is given by $-16x^2 + 40x$. When will the ball hit the ground?

10. The fuel economy in miles per gallon of a certain truck is given by the expression $-0.02x^2 + 1.5x + 3.4$, where x is the truck's speed in miles per hour. For what speed(s) does the truck have a fuel economy of 20 miles per gallon?

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Lesson 3.1.7: Applying the Quadratic Formula

Warm-Up 3.1.7

Sadie wants to sew a patch onto her shirt in the shape of an equilateral triangle with sides that are 4 inches long. The area of an equilateral triangle with sides of length *s* is given by $A=\frac{\sqrt{3}}{4}s^2$.

1. How many square inches of fabric will Sadie need to make her patch?

2. If Sadie decides to make 3 of her friends each a matching patch, how many square inches of fabric will Sadie need for the additional patches?

Scaffolded Practice 3.1.7

Example 1

Given the standard form of a quadratic equation, $ax^2 + bx + c = 0$, derive the quadratic formula by completing the square.

1. Begin with a quadratic equation in standard form.

2. Subtract c from both sides.

3. Divide both sides by a.

4. Complete the square.

5. Write the left side of the equation as a binomial squared and simplify the right side of the equation.

6. Take the square root of both sides of the equation and simplify the right side.

7. Subtract $\frac{b}{2a}$ from both sides of the equation to solve for x.

8. Combine the two fractions from the previous step.

continued

Example 2

Use the discriminant of $3x^2 - 5x + 1 = 0$ to identify the number and type of solutions.

Example 3

Solve $2x^2 - 5x = 12$ using the quadratic formula.

Example 4

Solve $x^2 = 2x - 1$ using the quadratic formula.

Example 5

Solve $5x^2 + 2x + 3 = 0$ using the quadratic formula.

Problem-Based Task 3.1.7: Divine Proportion

SMP	
1	2 ✓
3	4
5	6
7 ✓	8

The golden ratio is a special number. It is often represented by the Greek letter *phi*, ϕ. It is often suggested that artists and architects have long used the golden ratio to create work that is pleasing to the human eye. The ancient Greeks called the golden ratio the divine proportion; they believed the number to have mystical properties. One property of the golden ratio is that it is the only positive number that when increased by 1 is equal to its square. Find the golden ratio.

Practice 3.1.7: Applying the Quadratic Formula A

For problems 1 and 2, find the discriminant. Determine the number and type of roots of the equation.

1. $4x^2 + 4x + 1 = 0$

2. $x^2 + 3x = -2x - 6$

For problems 3–6, solve using the quadratic formula.

3. $-2x^2 + 3x + 4 = 0$

4. $16 - 8x - x^2 = 0$

5. $3x^2 + 7x + 12 = 0$

6. $-32x = 2x^2 - x - 51$

For problems 7–10, read each scenario and use the quadratic formula to answer the questions.

7. The height of a softball in meters x seconds after it has been thrown upward is given by $-4.9x^2 + 9x + 1.2$. After how many seconds does the ball hit the ground?

8. A company sells about $20x - x^2$ units each month, where x is the price of one unit. For what price(s) does the company sell 100 units?

9. As part of a science experiment, Carson designs and creates a cushioned egg carrier. He puts an egg inside it, and then drops it from a window to see whether his design can safely cushion the egg and keep it from breaking. The egg's height in feet x seconds after being dropped is given by $27 - 16x^2$. After how many seconds will the egg hit the ground?

10. How does the quadratic formula show the number and type of solutions of a quadratic equation?

Practice 3.1.7: Applying the Quadratic Formula B

For problems 1 and 2, find the discriminant. Write the number and type of roots of the equation.

1. $3x^2 - 5x + 1 = 0$

2. $-2x^2 - 4x = 12$

For problems 3–6, solve using the quadratic formula.

3. $x^2 + 2x + 1 = 0$

4. $3x^2 + 8x + 5 = 0$

5. $3x^2 - 7x + 14 = 0$

6. $-6x = 7x^2 - x - 12$

Read each scenario and use your knowledge of the quadratic formula to answer the questions.

7. The height of a golf ball in meters x seconds after it has been hit is given by $-4.9x^2 + 42x$. When does the ball hit the ground?

8. A girl downloads about $24x - x^2$ songs each month, where x is the price of one song. For what price(s) does the girl download 100 songs?

9. An apple falls from a tall branch. Its height in feet x seconds after it falls is given by $40 - 16x^2$. After how many seconds will the apple hit the ground?

10. Can a quadratic equation have one real solution and one non-real solution?

Notes

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Notes

Lesson 3.1.8: Solving Quadratic Inequalities

Warm-Up 3.1.8

A small company manufactures two types of computer games. One puzzle game takes 0.6 hour for assembly, and one racing game takes 0.3 hour for assembly. The company has at most 240 labor hours available for game assembly.

1. Let the number of puzzle games equal p and the number of racing games equal r. Write an inequality to model the situation.

2. If the company's manager decides the workers should assemble 100 puzzle games, how many racing games can be assembled?

Scaffolded Practice 3.1.8

Example 1

What is the solution of the inequality $(x - 2)(x + 10) > 0$?

1. Determine the sign possibilities for each factor.

2. Determine when both factors are positive.

3. Determine when both factors are negative.

continued

Example 2

Solve $x^2 + 8x + 7 \leq 0$. Graph the solution on a number line.

Example 3

Solve $4x - 1 > 8 - x^2$. Graph the solution on a number line.

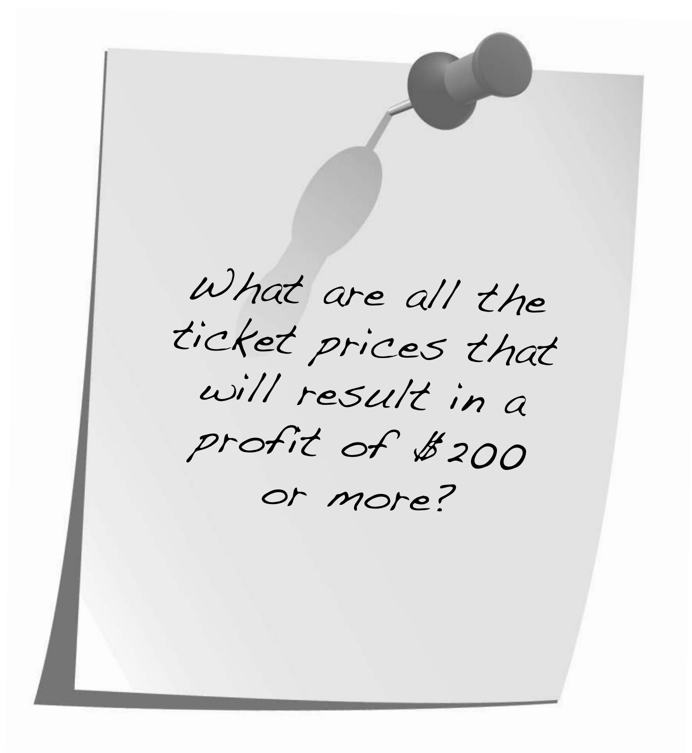
Example 4

What is the solution of $3x^2 + 2x + 2 < 0$?

Problem-Based Task 3.1.8: Dancing for Charity

SMP	
1	2 ✓
3	4
5 ✓	6 ✓
7	8

A school is planning to host a dance with all profits going to charity. The amount of profit is found by subtracting the total costs from the total income. The income from ticket sales can be expressed as $200x - 10x^2$, where x is the cost of a ticket. The costs of putting on the dance can be expressed as $500 + 20x$. What are all the ticket prices that will result in a profit of \$200 or more?



Practice 3.1.8: Solving Quadratic Inequalities **A**

For problems 1–7, solve each quadratic inequality. Graph the solution(s), if any, on a number line.

1. $(x + 3)(x - 2) < 0$

2. $(3x + 4)(2x - 1) \geq 0$

3. $x^2 - 25 \geq 0$

4. $x^2 + x - 12 < 0$

5. $x^2 + 20x - 3 \leq 0$

6. $x^2 + 12 \geq -7x$

7. $4x^2 + 4 < x$

For problems 8–10, solve each inequality using the given information.

8. A flying squirrel jumps from one tree to the next. Its height in feet x seconds into the jump is given by $-16x^2 + 32x + 4$. After how many seconds is the squirrel more than 12 feet above the ground?

9. Milo dives into a pool from a platform. His height above the water in feet x seconds into the dive is given by the expression $-5x^2 + 5x + 3$. After how many seconds is Milo more than 4 feet above the water?

10. Under certain conditions, the stopping distance of a vehicle in feet is given by $0.055x^2 + 1.1x$, where x is the speed of the vehicle in miles per hour. At what speeds is the stopping distance less than 40 feet?

Practice 3.1.8: Solving Quadratic Inequalities B

For problems 1–7, solve each quadratic inequality. Graph the solution(s), if any, on a number line.

1. $(2x + 1)(x - 1) \geq 0$

2. $(x - 2)(5x + 7) \leq 0$

3. $x^2 + 4 < 0$

4. $x^2 + 13x + 22 \geq 0$

5. $2x^2 + 16x - 3 > 0$

6. $x^2 + 10x - 3 < 7x$

7. $4x + 7 \leq 4x^2$

For problems 8–10, solve each inequality using the given information.

8. The height of a helicopter in meters x seconds after it takes off is given by $-4.9x^2 + 42x$. When is the helicopter more than 5 meters above the ground?

9. Livia drops a water balloon from her apartment window onto the ground below. The balloon's height above the ground in feet x seconds into the drop is given by the expression $-5x^2 + 5x + 4$. When is the balloon more than 6 feet above the ground?

10. The length of a pendulum in centimeters is given by the expression $\frac{9.8x^2}{4\pi^2}$, where x is the time in seconds for the pendulum to swing from one side to the other. When the length of the pendulum is greater than 5 centimeters, what can you say about the time it takes for the pendulum to swing from one side to the other?

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Lesson 3.2.1: Creating and Graphing Equations Using Standard Form

Warm-Up 3.2.1

The point guard for your school's basketball team shoots the ball from 11.5 feet away from the basket. As the table shows, the height of the ball changes as it nears the basket.

Distance from basket in feet (x)	**Ball height in feet (y)**
11.5	6
10	10.4
9	11.8
5	14.3
4	13.8
0	10

1. Sketch a graph to show the relationship between the distance the ball is from the basket and the height of the ball. Connect the points with a curve.

2. About how high do you think the ball will be after traveling a horizontal distance of 8 feet from the point guard toward the basket? Explain your reasoning.

Scaffolded Practice 3.2.1

Example 1

Find the y-intercept and vertex of the function $f(x) = -2x^2 + 4x + 3$. Determine whether the vertex is a minimum or maximum point on the graph.

1. Determine the y-intercept.

2. Determine the vertex of the function.

3. Use the value of a to determine if the vertex is a maximum or a minimum value.

continued

Example 2

$h(x) = 2x^2 - 11x + 5$ is a quadratic function. Determine the direction in which the function opens, the coordinates of the vertex, the axis of symmetry, the x-intercept(s), if any, and the y-intercept. Use this information to sketch the graph.

Example 3

$R(x) = 2x^2 + 8x + 8$ is a quadratic function. Determine the direction in which the function opens, the vertex, the axis of symmetry, the x-intercept(s), if any, and the y-intercept. Use this information to sketch the graph.

Example 4

$g(x) = -x^2 + 8x - 17$ is a quadratic function. Determine the direction in which the function opens, the vertex, the equation of the axis of symmetry, the x-intercept(s), if any, and the y-intercept. Use this information to sketch the graph.

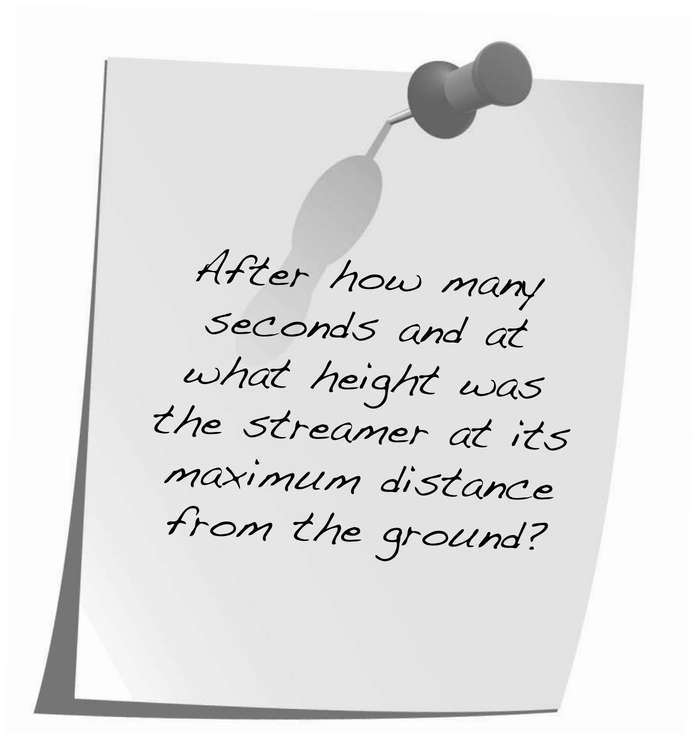
Example 5

Create the equation of a quadratic function given a vertex of (2, –4) and a y-intercept of 4.

Problem-Based Task 3.2.1: Parabolic Party Streamers

SMP	
1 ✓	2
3	4 ✓
5	6
7 ✓	8

Students in a physics class are testing the pull of gravity at varying heights. With permission of the building manager, each student went to a different floor of a tall office building and tossed a roll of paper streamer up into the air from the window. Another student videotaped the streamers' paths downward so that the class could determine the approximate equations of the parabolas the streamers created as they unraveled. When a streamer was thrown upward from the highest story of the building, the students determined that the distance, in feet, between the streamer and the ground t seconds after the streamer was thrown could be expressed by $h(t) = -16t^2 + 32t + 56$. After how many seconds and at what height was the streamer at its maximum distance from the ground?



Practice 3.2.1: Creating and Graphing Equations Using Standard Form A

Sketch the graph for each of the following quadratic functions.

1. $q(x) = -x^2 - 6x - 8$

2. $f(x) = -3x^2 + 24x - 48$

3. $m(b) = b^2 - 6b + 10$

Find the y-intercept and vertex of the following functions. State whether the vertex is a minimum or maximum point on the graph and explain your reasoning.

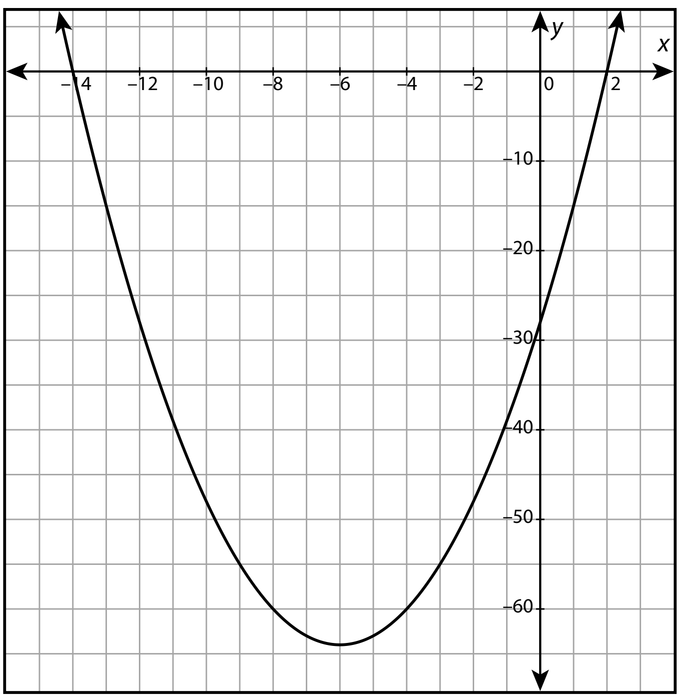
4. $k(h) = h^2 - 4h + 3$

5. $l(d) = d^2 - 6d$

6. $f(x) = -7x^2 - 14x - 6$

Does the following graph represent the given function? Explain your reasoning.

7. $y(x) = x^2 + 12x - 28$



continued

Use your knowledge of quadratic functions to complete the problems that follow.

8. Create the equation of a quadratic function with a vertex of (–3, 7) and a *y*-intercept of –2.

9. The path of an arrow shot in the air can be modeled by the function $h(t) = -16t^2 + 144t + 4$, where *h* is the height, in feet, of the arrow above ground *t* seconds after it is released. Determine the maximum height that the arrow reaches.

10. The demand, *d*, for plastic storage containers depends on their price. A retail manager determines that the number of containers she can sell at a price of *x* dollars each is given by the formula $d(x) = -3x^2 + 220x - 200$. At what price will the demand for the containers be at a maximum?

Practice 3.2.1: Creating and Graphing Equations Using Standard Form B

Sketch the graph for each of the following quadratic functions.

1. $a(x) = 2x^2 - 6x + 4$

2. $e(x) = x^2$

3. $f(x) = x^2 + 2$

Find the y-intercept and vertex of the following functions. State whether the vertex is a minimum or maximum point on the graph and explain your reasoning.

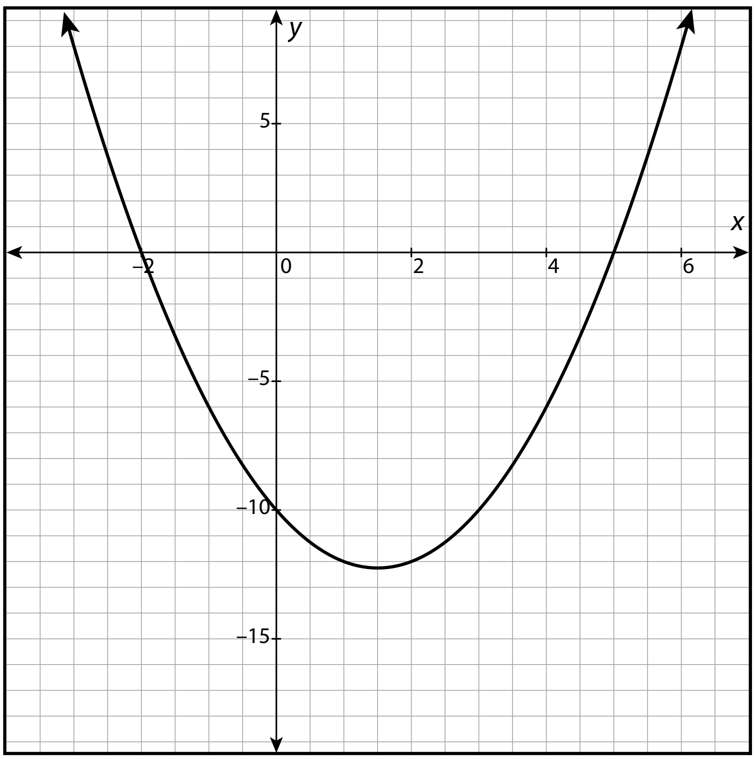
4. $n(h) = -2h^2 - 7h$

5. $l(r) = 4r^2 + 40r + 7$

6. $f(x) = -2x^2 + 4x + 3$

Does the following graph represent the given function? Explain your reasoning.

7. $d(t) = t^2 - 3t - 5$



continued

Use your knowledge of quadratic functions to complete the problems that follow.

8. Create the equation of a quadratic function with a vertex of (5, 6) and a *y*-intercept of –69.

9. The path of a ball shot up in the air from a slingshot can be modeled by the function $h(t) = -16t^2 + 150t + 4$, where *h* is the height, in feet, of the ball above ground *t* seconds after it is released. Determine the maximum height that the ball reaches, rounded to the nearest foot.

10. A sock manufacturing company's profit *p* (in hundreds of dollars) after selling *x* thousand pairs of socks can be modeled by the function $p(x) = -4x^2 + 40x - 2$. How many pairs of socks must be sold in order to maximize profits?

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Lesson 3.2.2: Creating and Graphing Equations Using the *x*-intercepts

Warm-Up 3.2.2

Peyton is the top car salesperson at her dealership. She makes a fixed salary of $2,500 per month, plus a 0.2% commission on sales.

1. Write an equation to represent Peyton's monthly take-home pay.

2. Graph the equation.

3. What are the x- and y-intercepts?

4. What do the x- and y-intercepts mean in terms of Peyton's monthly take-home pay?

Scaffolded Practice 3.2.2

Example 1

Determine the equation of a quadratic function in standard form, given the zeros $x = 2$ and $x = -2$, and the point (0, 3) that lies on the graph of the function.

1. Write the zeros as expressions equal to 0 to determine the factors of the quadratic equation.

2. Use the point (0, 3) to find the value of a.

3. Write the equation in standard form.

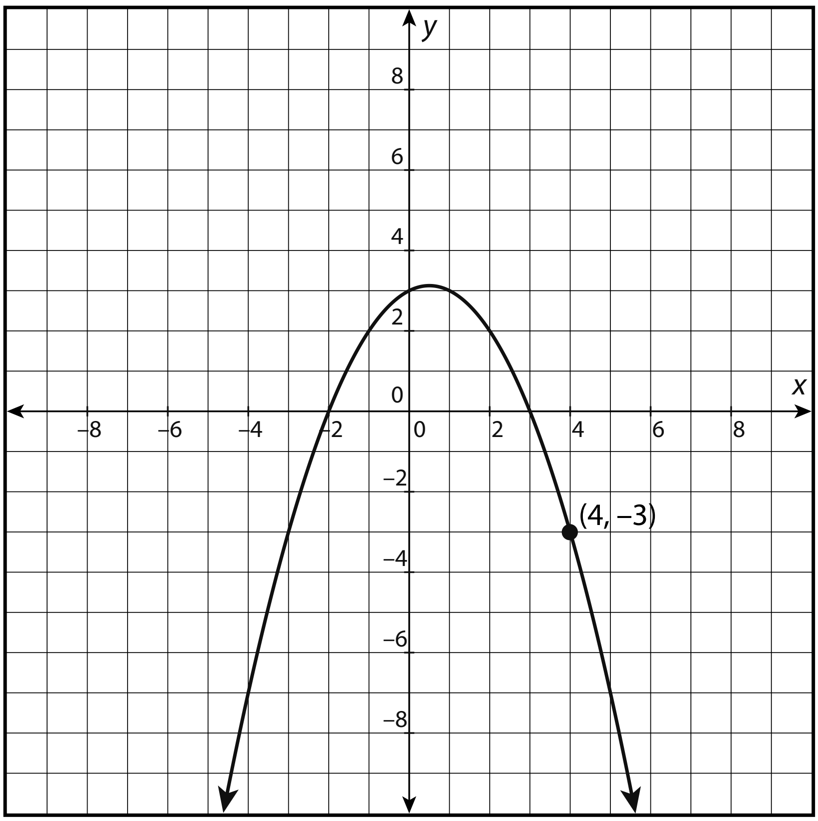
continued

Example 2

Identify the x-intercepts, if any, the equation of the axis of symmetry, and the vertex of the quadratic function $f(x) = (x + 5)(x + 2)$. Use this information to graph the function.

Example 3

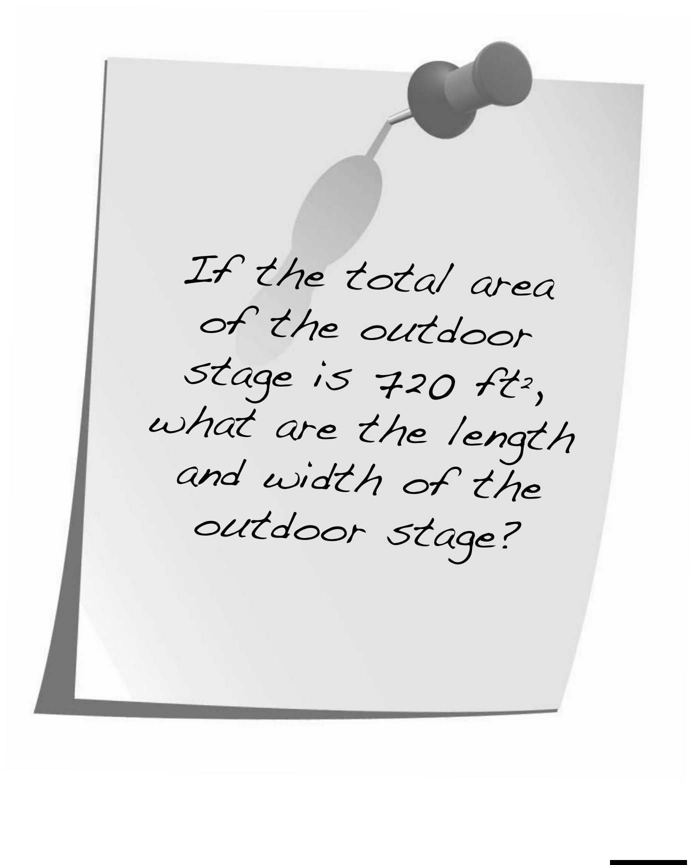
Use the x-intercepts and the graphed point to write the equation of the function in standard form.



Problem-Based Task 3.2.2: It's Show Time!

SMP	
1	2 ✓
3	4
5	6 ✓
7 ✓	8

The school's drama department is putting on a production. Instead of using the school's indoor stage, the department head decided to build an outdoor stage to accommodate a greater audience. The width of the outdoor stage is 6 feet less than its length. If the total area of the outdoor stage is 720 ft^2, what are the length and width of the outdoor stage?



Practice 3.2.2: Creating and Graphing Equations Using the *x*-intercepts A

Identify the *x*-intercepts, if any, of the following quadratic functions. Determine the equation of the axis of symmetry for each parabola.

1. $y = (x - 3)(x + 6)$

2. $f(x) = \left(x - \frac{2}{3}\right)\left(x + \frac{2}{3}\right)$

Determine the equation of each quadratic function in standard form, given the zeros and a point on the graph.

3. $x = -2$; $(3, 10)$

4. $x = 5$, $x = -12$; $(0, -60)$

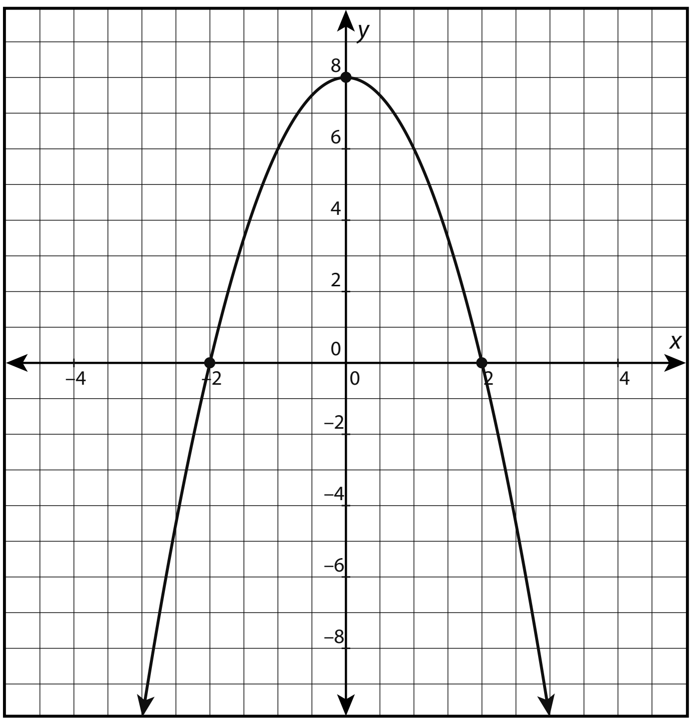
Sketch a graph for each of the following quadratic functions.

5. $y = (x + 3)(x + 1)$

6. $y = (3x - 2)(x - 1)$

Given the graph of a quadratic function, use the intercepts and another point on the graph to write the equation of the function in standard form.

7.



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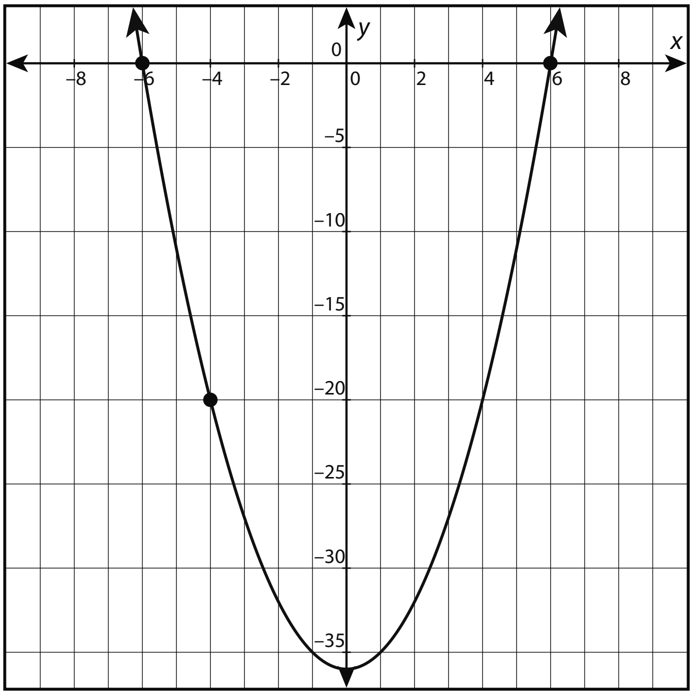
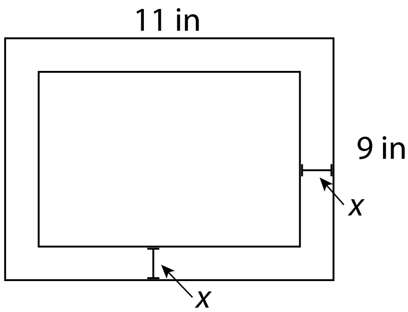
8.



Use the given information to solve the following problems.

9. A family portrait hanging on the wall has a frame with dimensions of 11 inches by 9 inches. The width of the frame is represented by x. What are the dimensions of the portrait if its area is 35 square inches?



10. A bird takes off from the roof of a 250-foot-tall building and flies to the ground below. Its path takes the form of a parabola. The bird's height can be modeled by $h(t) = -t^2 + 15t + 250$, where $h(t)$ is the height of the bird above ground in feet t seconds after leaving the roof. After how many seconds does the bird land on the ground?

Practice 3.2.2: Creating and Graphing Equations Using the *x*-intercepts B

Identify the *x*-intercepts, if any, of the following quadratic functions. Determine the equation of the axis of symmetry for each parabola.

1. $h(t) = (-16t + 1)(t - 7)$

2. $y = 2\left(x - \frac{3}{4}\right)\left(x + \frac{7}{2}\right)$

Determine the equation of each quadratic function in standard form, given the zeros and a point on the graph.

3. $x = -4$, $x = -2$; $(-3, -1)$

4. $x = 15$, $x = 5$; $(0, 75)$

Sketch a graph for each of the following quadratic functions.

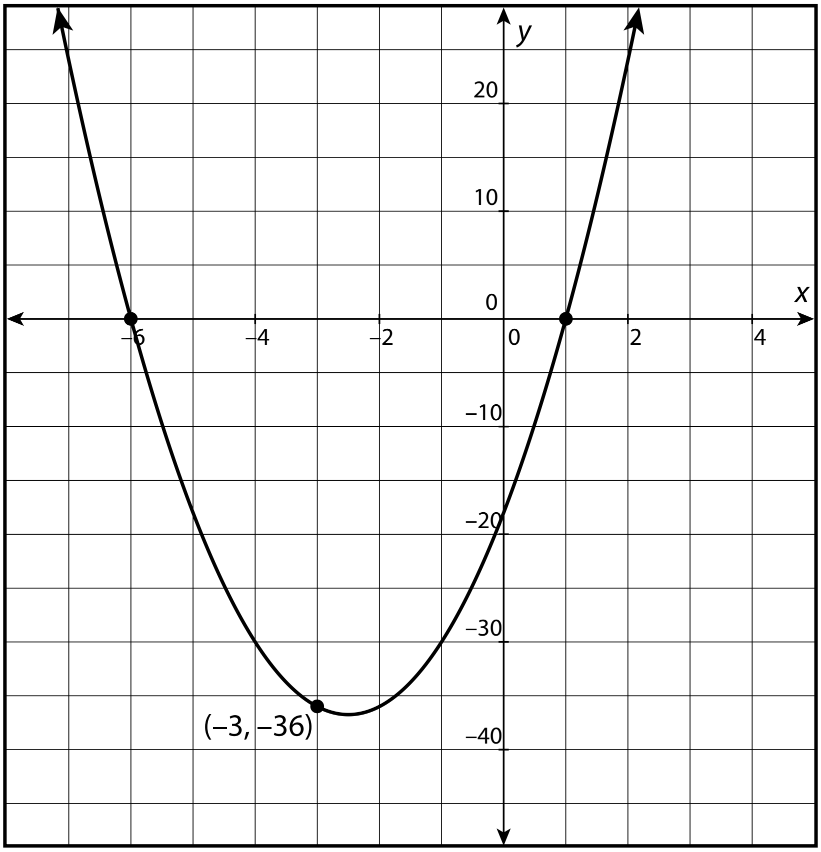
5. $f(x) = (x - 3)(x - 4)$

6. $g(x) = (x - 3)(x - 2)$

continued

Given the graph of a quadratic function, use the intercepts and a point to write the equation of the function in standard form.

7.



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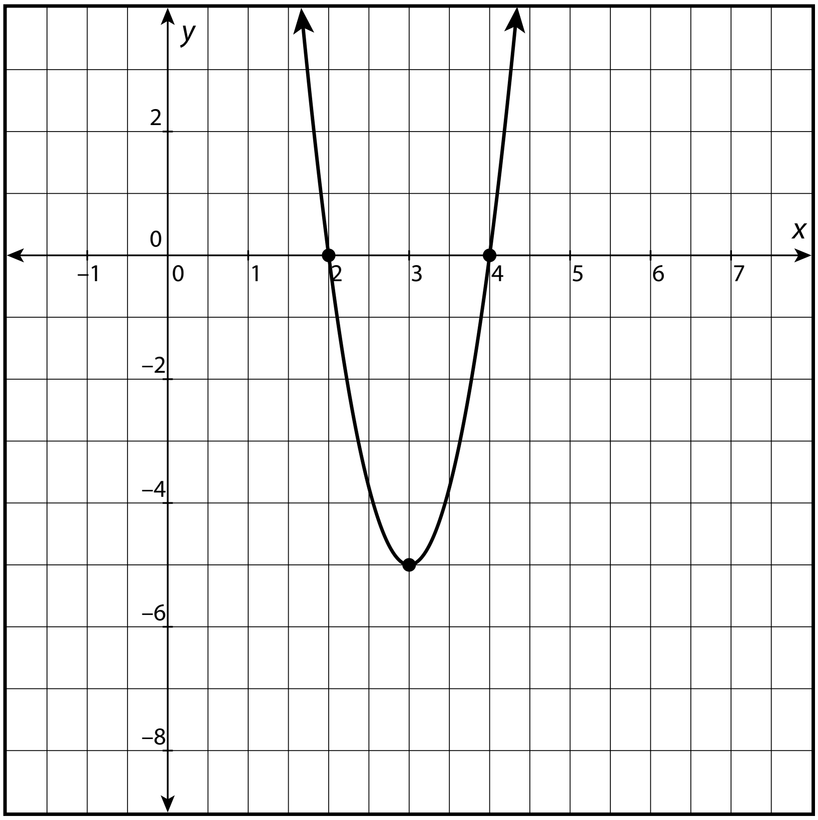
8.



Use the given information to solve the following problems.

9. A walkway is being installed around a rectangular playground. The playground is 30 feet by 12 feet, and the total area of the playground and the walkway is 1,288 ft². What is the width of the walkway?

10. A high school senior vacationing in Negril, Jamaica, for her senior trip jumped off a 20-foot cliff into a pool of water. The height of the senior above the water is modeled by the function $h(t) = -t^2 + \frac{1}{4}t + \frac{5}{4}$, where $h(t)$ is the height of the senior above the water in feet t seconds after jumping off the cliff. How many seconds will it take for the senior to reach the water?

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Lesson 3.2.3: Creating and Graphing Equations Using Vertex Form

Warm-Up 3.2.3

An ice-skating rink manager tracks each day's revenue *R* based on an hourly fee in dollars, *x*, for skating using the function $R(x) = -480x^2 + 3120x$.

1. What hourly fee will produce the maximum revenue?

2. What is the maximum revenue that the manager should expect?

Scaffolded Practice 3.2.3

Example 1

Given the quadratic function $f(x)=\frac{1}{2}(x+6)^2-2$, identify the vertex and determine whether it is a minimum or maximum.

1. Identify the vertex.

2. Determine if the vertex is a minimum or maximum.

continued

Example 2

Determine the equation of a quadratic function that has a minimum at (–4, –8) and passes through the point (–2, –5).

Example 3

Convert the function $g(x) = -7x^2 + 14x$ to vertex form.

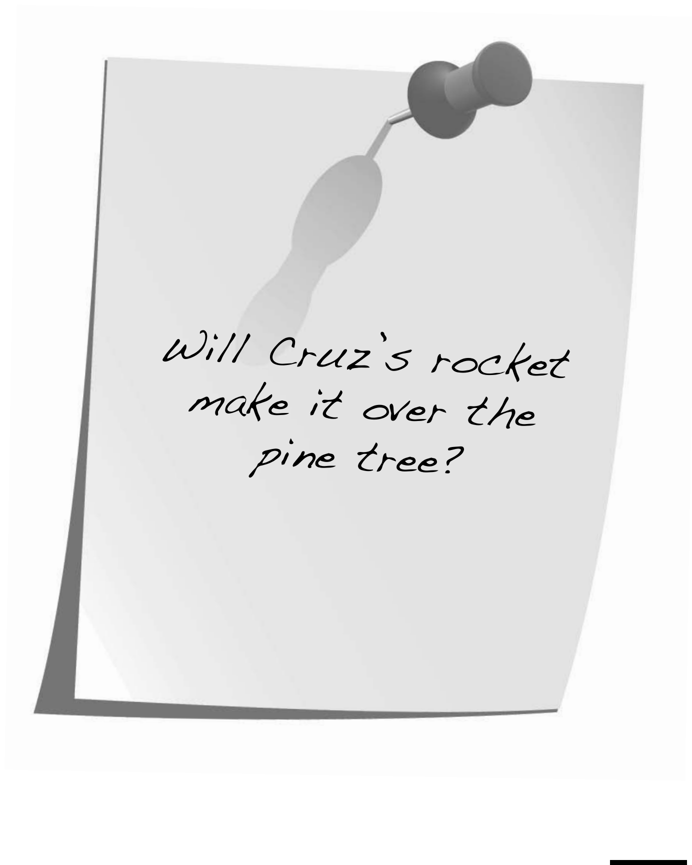
Example 4

Sketch a graph of the quadratic function $y = (x + 3)^2 - 8$. Label the vertex, the axis of symmetry, the y-intercept, and one pair of symmetric points.

Problem-Based Task 3.2.3: Rocket Heights

SMP	
1 ✓	2
3	4 ✓
5	6 ✓
7	8

The path of a model rocket launched from the ground is parabolic. Cruz built a reusable model rocket using a kit. After launching it a few times from level ground, Cruz determined that the rocket could reach a maximum height of 400 feet after traveling a horizontal distance of 15 feet. On the flight where it reached the maximum height, the rocket landed 30 feet away from the launch site. Cruz's friends asked him to launch the rocket again in an area with several trees. He's worried that the rocket will be destroyed if the trees block its path—especially since a 115-foot-tall pine tree stands just 2 feet from the landing spot. Will Cruz's rocket make it over the pine tree?



Practice 3.2.3: Creating and Graphing Equations Using Vertex Form A

For each of the following quadratic functions, identify the vertex, state whether the vertex is a minimum or a maximum, and explain your reasoning.

1. $y = -2(x + 3)^2 - 3$

2. $f(x) = (x - 6)^2 + 6$

Determine the equation of a quadratic function that satisfies the given criteria.

3. The coordinates of the vertex are (–2.5, –3) and it contains the point (–1, –1).

4. The coordinates of the vertex are (2, 10) and it contains the point (1, 8).

Convert each quadratic function given in standard form to vertex form.

5. $f(x) = x^2 + 6x + 9$

6. $g(x) = 3x^2 - 12x + 9$

Sketch a graph of each quadratic function. Label the vertex, the axis of symmetry, the *y*-intercept, and one pair of symmetric points.

7. $y = -2x^2 + 8$

8. $t(x) = -2(x - 2)^2 + 5$

Create the equation of a quadratic function with the given characteristics.

9. The vertex is at (24, 100) and the *x*-intercept is 48.

10. The vertex is at (3, 18) and the *x*-intercept is 6.

Practice 3.2.3: Creating and Graphing Equations Using Vertex Form B

For each of the following quadratic functions, identify the vertex, state whether the vertex is a minimum or a maximum, and explain your reasoning.

1. $y = -3(x + 4)^2 + 1$

2. $f(x) = 4(x - 2)^2$

Determine the equation of a quadratic function that satisfies the given criteria.

3. The coordinates of the vertex are (5.5, –6) and it contains the point (3, 0).

4. The coordinates of the vertex are (2, 0) and it contains the point (0, 20).

Convert each quadratic function given in standard form to vertex form.

5. $f(x) = x^2 - 2x - 2$

6. $g(x) = 0.3x^2 + 1.2x + 1.2$

Sketch a graph of each quadratic function. Label the vertex, the axis of symmetry, and one pair of symmetric points.

7. $y = (x + 4)^2 + 2$

8. $u(x) = -(x - 5)^2 - 2.5$

Create the equation of a quadratic function with the given characteristics.

9. The vertex is at (6, 72) and the x-intercept is 12.

10. The vertex is at (10.5, 385) and the x-intercept is 21.

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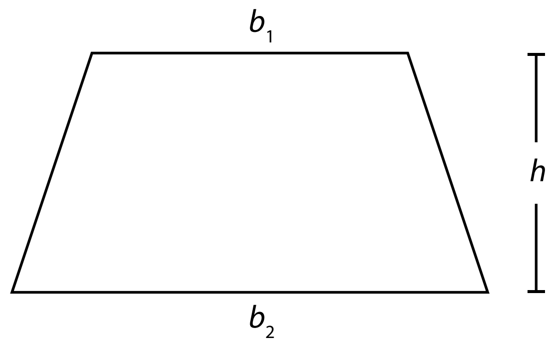
Lesson 3.2.4: Rearranging Formulas Revisited

Warm-Up 3.2.4

The walls of a greenhouse are shaped like a trapezoid. The area of a trapezoid is expressed by the formula $A=\frac{1}{2}(b_1+b_2)h$, where A is the area of the trapezoid, b_1 and b_2 are the lengths of the bases of the trapezoid, and h is the height.



1. If you know the area and the length of the two bases of one greenhouse wall, how could you find the height of this wall?

2. What is the formula that expresses the height of the wall?

3. If the two bases measure 30 feet and 34 feet respectively, and the area of the wall is 960 ft^2, what is the wall's actual height?

Scaffolded Practice 3.2.4

Example 1

Solve the equation $x^2 + y^2 = 100$ for y.

1. Isolate y.

2. Summarize your result.

continued

Example 2

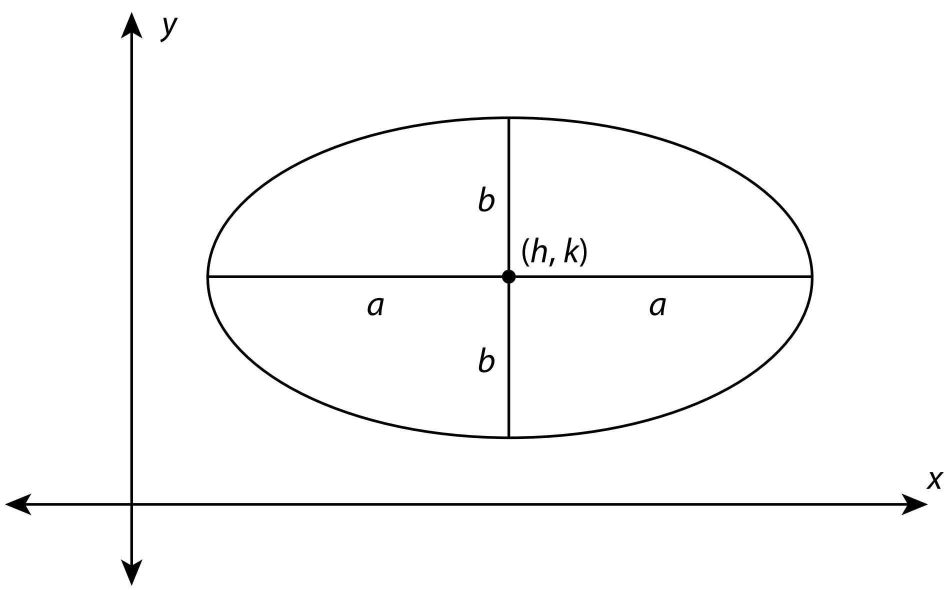
Solve $y = 3(x - 7)^2 + 8$ for x in terms of y.

Example 3

The formula for the area of a square is $A = s^2$, where s is the length of a side of the square. Solve the formula for s.

Example 4

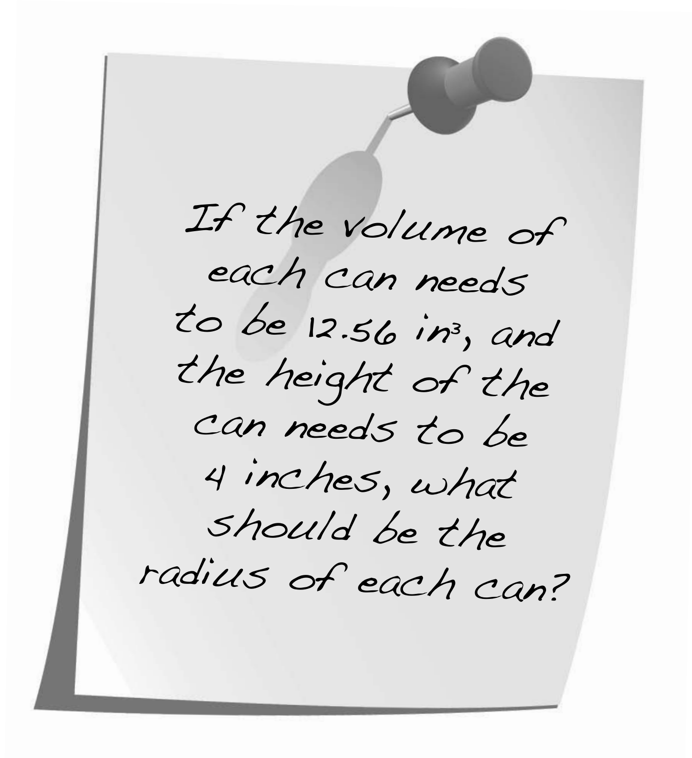
The equation to graph an ellipse is $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, where (h, k) is the center of the ellipse, a is the number of units right and left of the center on the ellipse, and b is the number of units up and down from the center on the ellipse. Solve the formula for a.



Problem-Based Task 3.2.4: Selecting a Cylinder Size

SMP	
1	2 ✓
3	4 ✓
5	6
7 ✓	8

A local soda company is investigating several different models for a cylindrical, recyclable can for its organic root beer. The formula for the volume of a cylinder is $V = \pi r^2 h$, where πr^2 is the area of the cylinder's base and h is the height of the cylinder. If the volume of each can needs to be 12.56 in^3, and the height of the can needs to be 4 inches, what should be the radius of each can?



Practice 3.2.4: Rearranging Formulas Revisited **A**

For problems 1–4, solve each equation for *x* in terms of *y*.

1. $x^2 + y^2 = 25$

2. $y = (x + 3)^2$

3. $(x+7)^2 = \frac{49y^2}{16}$

4. $2(x + 1)^2 - 90 = y$

Problems 5–10 each describe a different mathematical formula. Solve each problem for the requested variable.

5. The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$, where *r* is the radius of the base and *h* is the height. Solve for the radius of the base.

6. The formula for the area of a circle is $A = \pi r^2$, where *A* is the area and *r* is the radius. Solve for the radius.

7. The formula $a^2 + b^2 = c^2$ is the Pythagorean Theorem, where *c* is the length of the hypotenuse of a right triangle and *a* and *b* are the lengths of the two legs. Rearrange this formula so that the focus is *b*.

8. Kinetic energy is the energy of a moving object. The formula to determine the kinetic energy of an object is $E_k = \frac{1}{2}mv^2$, where E_k is the kinetic energy, *m* is the mass of the object, and *v* is the object's speed. Solve this formula for the object's speed, assuming that kinetic energy is positive.

9. The vertex form of a quadratic is $y = a(x - h)^2 + k$. Solve this equation for *x*.

10. The formula to graph an ellipse is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, where (h, k) is the center of the ellipse, *a* is the number of units right or left from the center, and *b* is the number of units up or down from the center. Solve the formula for *b*.

Practice 3.2.4: Rearranging Formulas Revisited B

For problems 1–4, solve each equation for *x* in terms of *y*.

1. $x^2 + 4y^2 = 360$

2. $y = 16x^2 - 9$

3. $y = 4(x + 7)^2$

4. $2.5(x - 6.5)^2 - 20 = y$

Problems 5–10 each describe a different mathematical formula. Solve each problem for the requested variable.

5. The formula to determine the power of an electrical charge is $P = \frac{V^2}{R}$, where *P* is the power, *V* is the electric potential difference (a value that can be negative), and *R* is the resistance. Solve this equation for the electric potential difference.

6. The area of a sector of a circle is represented by $A = \frac{1}{2}r^2\theta$, where *A* is the area of the sector, *r* is the radius, and θ is the angle created by the sides of the sector and the center of the circle. Solve for the radius.

7. The formula $a^2 + b^2 = c^2$ is the Pythagorean Theorem, where *c* is the length of the hypotenuse of a right triangle and *a* and *b* are the lengths of the other two legs. Rearrange this formula so that the focus is *c*.

continued

8. In physics, the circular motion of an object can be defined by the formula $a = \frac{v^2}{r}$, where a is the centripetal acceleration (directed towards the center of the circle), v is the tangential velocity, and r is the radius. Solve for the tangential velocity.

9. The formula for a circle drawn in the coordinate plane is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle and r is the radius. Solve for y.

10. The formula to find the surface area of a sphere is $SA = 4\pi r^2$, where SA is the surface area and r is the radius of the sphere. Solve for the radius.

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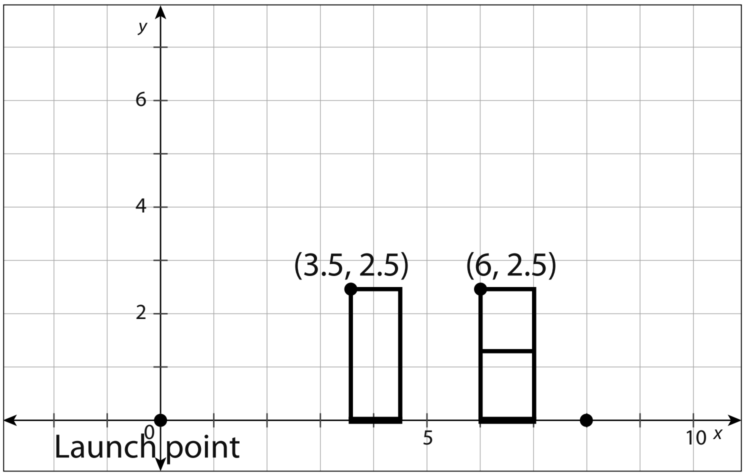
Lesson 3.3.1: Interpreting Key Features of Quadratic Functions

Warm-Up 3.3.1

The object of a popular video game is to launch a boulder to knock over boxes, buildings, and other items. The graph shows an obstacle on the left that the boulder must clear in order to knock over the stack of boxes on the right. The boulder will follow a parabolic path and will launch from (0, 0) and end at (8, 0).



1. What are the x-intercepts for the parabola formed by the path of the boulder?

2. What is the axis of symmetry for the parabola formed by the path of the boulder? How do you know?

3. One possible path for the boulder is $y=-\frac{3}{8}x^2+3x$. What is the vertex of the parabola created by this equation?

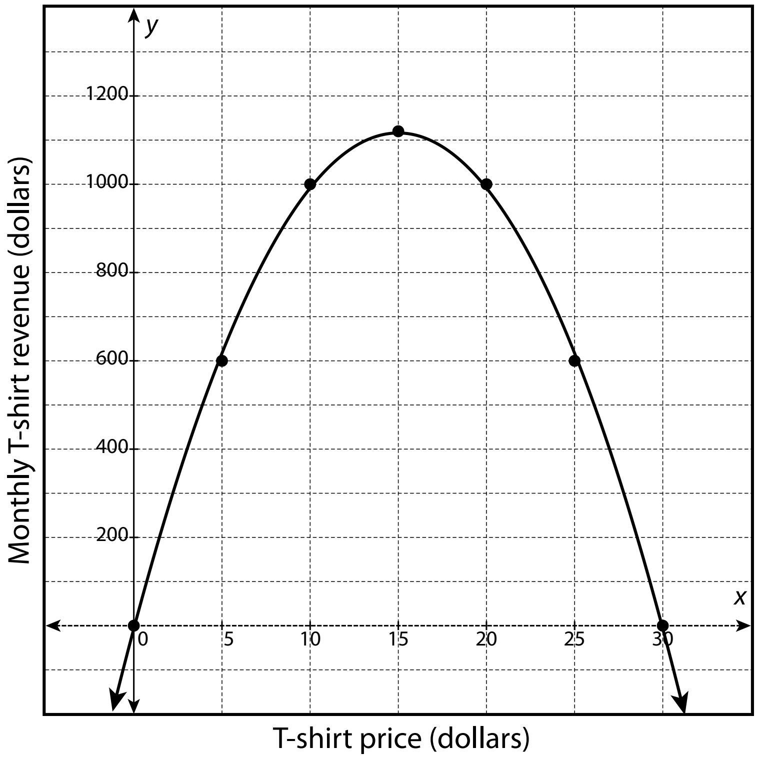
4. Will the boulder clear the obstacle? How do you know?

5. Will the boulder knock down the boxes? How do you know?

Scaffolded Practice 3.3.1

Example 1

A local store's monthly revenue from T-shirt sales, $f(x)$, as a function of price, x, is modeled by the function $f(x) = -5x^2 + 150x - 7$. Use the equation and graph to answer the following questions: At what prices is the revenue increasing? Decreasing? What is the maximum revenue? What prices yield no revenue? Is the function even, odd, or neither?



1. Determine when the function is increasing and decreasing.

2. Determine the maximum revenue.

3. Determine the prices that yield no revenue.

4. Determine if the function is even, odd, or neither.

5. Use the graph of the function to verify that the function is neither odd nor even.

continued

Example 2

A quadratic function has a minimum value of –5 and *x*-intercepts of –8 and 4. What is the value of *x* that minimizes the function? For what values of *x* is the function increasing? Decreasing?

Example 3

The table shows the predicted temperatures for a summer day in Woodland, California. At what times is the temperature increasing? Decreasing?

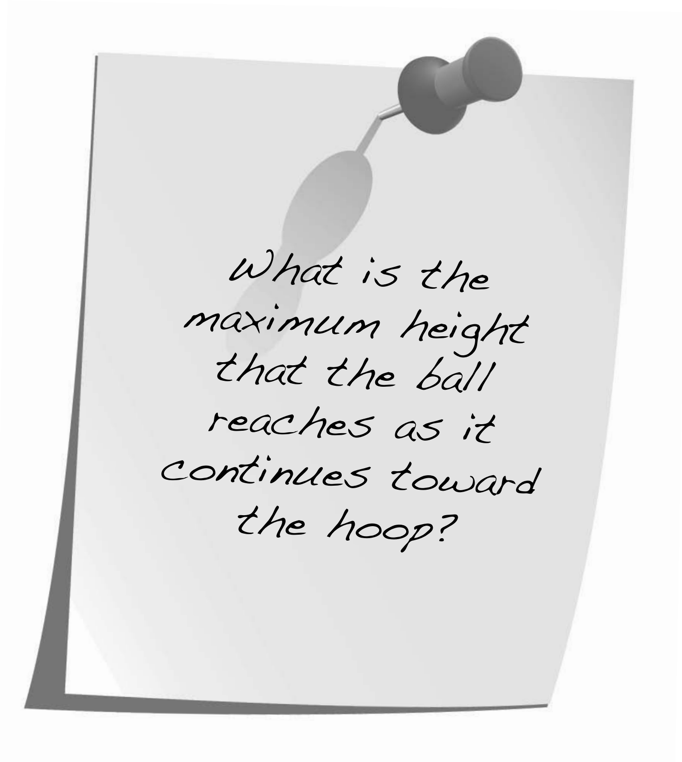
Time	Temperature (°F)
8 A.M.	52
10 A.M.	64
12 P.M.	72
2 P.M.	78
4 P.M.	81
6 P.M.	76

Problem-Based Task 3.3.1: One-on-One Basketball

SMP
1 ✓ 2
3 4 ✓
5 6 ✓
7 8

You and a friend are playing one-on-one basketball at the park. You aim at the hoop and release the ball, which follows a parabolic path. The table represents the ball's horizontal distance from you and the ball's height as it travels toward the center of the hoop, which is represented by the point (14, 10). Use a quadratic model to determine for what horizontal distances the height of the ball is increasing and decreasing. What is the maximum height that the ball reaches as it continues toward the hoop?

Distance from shooter (feet)	Height of basketball (feet)
4	10
6	12
12	12
14	10



Practice 3.3.1: Interpreting Key Features of Quadratic Functions A

For each of the given quadratic functions, use graphing technology to answer the following questions: What are the *x*-values for which the function is increasing? Decreasing? What is the maximum or minimum value of the function? What are the intercepts? Is the function even, odd, or neither?

1. $f(x) = 3x^2 - 2x - 5$

2. $g(x) = -3x^2 + 10x + 1$

3. $y = 5x^2 + 10x + 11$

4. $h(x) = 2x^2 - 4x - 11$

Given the following descriptions of quadratic functions, answer the questions: What is the value of *x* that minimizes or maximizes the function? For what values of *x* is the function increasing? Decreasing?

5. A function has a minimum value of –20 and *x*-intercepts of –1.72 and 0.38.

6. A function has a maximum value of 12.375 and *x*-intercepts of 0.41 and 1.84.

7. A function has a minimum value of –8.675 and *x*-intercepts of 1.23 and –0.48.

8. A function has a minimum value of –8.167 and *x*-intercepts of 1.3 and –1.

continued

Use the tables and scenarios that follow to complete the remaining problems.

9. You and a friend are playing softball. You throw the ball toward your friend's mitt so that the ball follows a parabolic path. The table represents the ball's horizontal distance from you and the ball's height as it travels from a starting position of (0, 4). Use a quadratic model to determine for what distances the height of the ball is increasing and decreasing.

Distance from you (feet)	Height of softball (feet)
0	4
20	10
30	10
40	8

10. The table shows the predicted temperatures for an autumn day in Annapolis, Maryland. Use a quadratic model to determine the maximum temperature that Annapolis reaches on this day.

Time	Temperature (°F)
11 A.M.	59
2 P.M.	63
5 P.M.	63
8 P.M.	58
11 P.M.	56

Practice 3.3.1: Interpreting Key Features of Quadratic Functions B

For each of the given quadratic functions, use graphing technology to answer the following questions: What are the *x*-values for which the function is increasing? Decreasing? What is the maximum or minimum value of the function? What are the *x*-intercepts? Is the function even, odd, or neither?

1. $f(x) = x^2 - 3x - 6$

2. $g(x) = -x^2 - 4x + 7$

3. $y = -4x^2 + 8x + 12$

4. $h(x) = 5x^2$

Given the following descriptions of quadratic functions, answer the questions: What is the value of *x* that minimizes or maximizes the function? For what values of *x* is the function increasing? Decreasing?

5. A function has a minimum value of –16.3 and *x*-intercepts of 9.3 and –41.9.

6. A function has a minimum value of –8.125 and *x*-intercepts of 4.27 and 0.23.

7. A function has a maximum value of 0.417 and *x*-intercepts of –1.618 and 0.618.

8. A function has a minimum value of –1.02 and *x*-intercepts of –0.165 and 0.124.

continued

Use the tables and scenarios that follow to complete the remaining problems.

9. You are practicing punting the football before football tryouts. You kick the ball from the ground represented by the point (0, 0), and the path of the ball is parabolic. The table represents the height of the ball seconds after being kicked. Use a quadratic model to determine at what times the height of the ball is increasing and decreasing.

Time (seconds)	Height of football (feet)
0	0
0.5	12
1	16
1.5	12
2	0

10. The table shows the height of a signal flare seconds after it is shot from the deck of a ship. Signal flares explode when they reach their highest point. Use a quadratic model to determine how high the flare will be when it explodes.

Time (seconds)	Height of flare (feet)
0	112
1	192
2	240
3	256
4	240
5	192

Notes

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Notes

Lesson 3.3.2: Identifying the Domain and Range of a Quadratic Function

Warm-Up 3.3.2

Gina works in a video game store over her summer break. She earns $8 per hour plus commission on the video games and gaming consoles she sells. This month, her store offered an incentive of an extra day off to the employee who sells the most copies of a certain new game. She makes $3 in commission for each game sold.

1. Write a linear model to represent Gina's take-home pay as a function of the number of games she sells in one 8-hour workday.

2. What is a reasonable domain for this function?

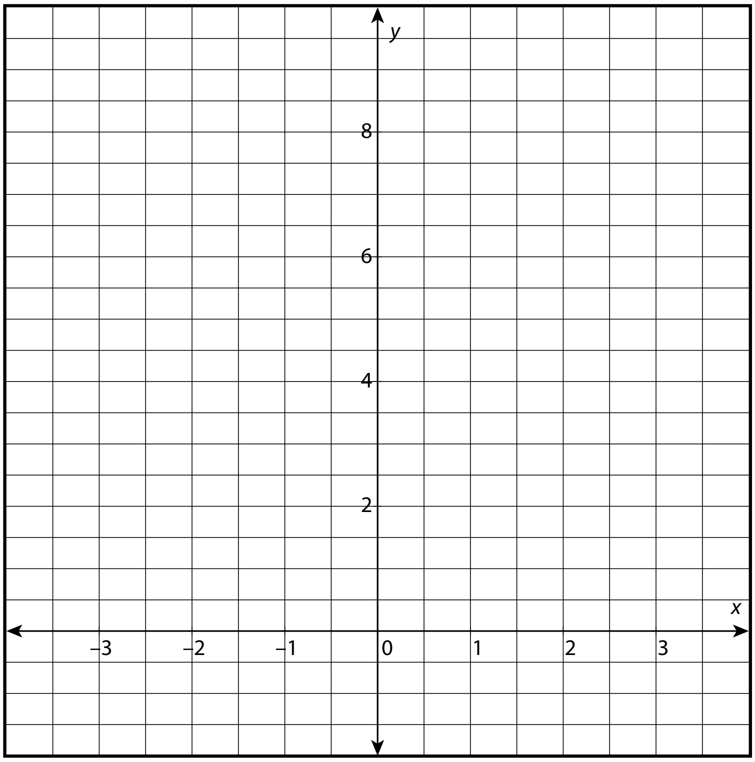
3. What does the domain represent?

Scaffolded Practice 3.3.2

Example 1

Describe the domain of the quadratic function $g(x) = 1.5x^2$.

1. Sketch a graph of the function.

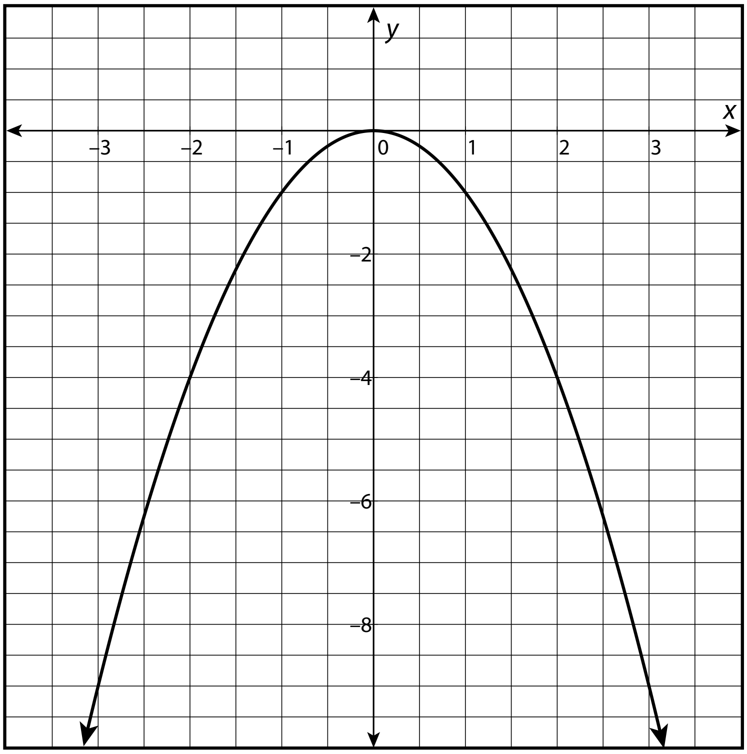


2. Describe what will happen if the function continues.

continued

Example 2

Describe the domain of the following function.



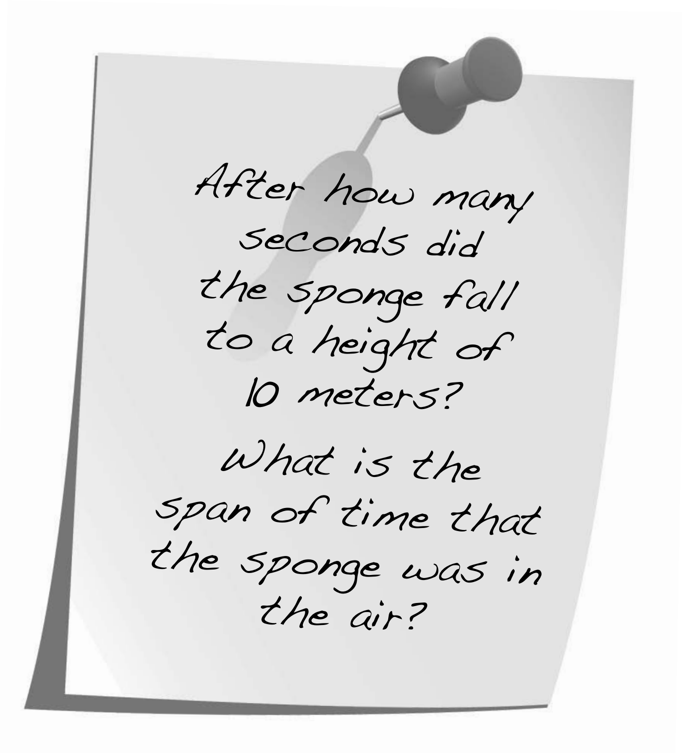
Example 3

Amit is a diver on the swim team. Today he's practicing by jumping off a 14-foot platform into the pool. Amit's height in feet above the water is modeled by $f(x) = -16x^2 + 14$, where x is the time in seconds after he leaves the platform. About how long will it take Amit to reach the water? Describe the domain of this function.

Problem-Based Task 3.3.2: Window Washers

A window washer tossed a wet sponge from a height of 10 meters above ground level to his coworker above him. The sponge reached its maximum height of 11.25 meters exactly 0.5 second later, but the coworker did not catch the sponge and it fell to the ground. After how many seconds did the sponge fall to a height of 10 meters? What is the span of time that the sponge was in the air?

SMP	
1 ✓	2 ✓
3	4
5	6 ✓
7	8



Practice 3.3.2: Identifying the Domain and Range of a Quadratic Function A

Use graphing technology to determine the domain and range of each quadratic function.

1. $y = -x^2 + 7x + 1$

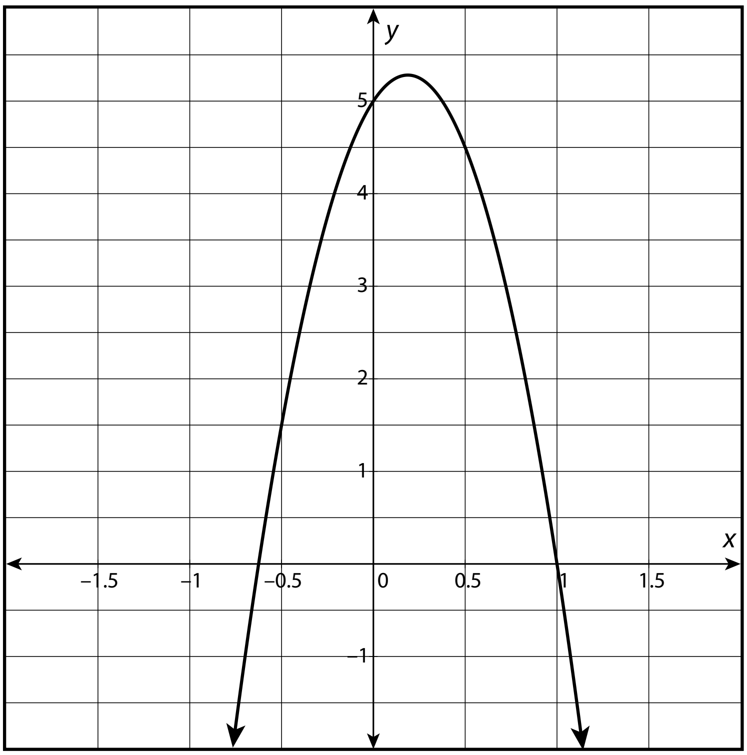
2. $y = -\frac{3}{5}x^2 + 21x - 3$

3. $f(x) = 4x^2 + 5x - 12$

4. $g(x) = x^2 + 12x - 8$

Describe the domain and range of each of the following functions in words and as an inequality.

5.



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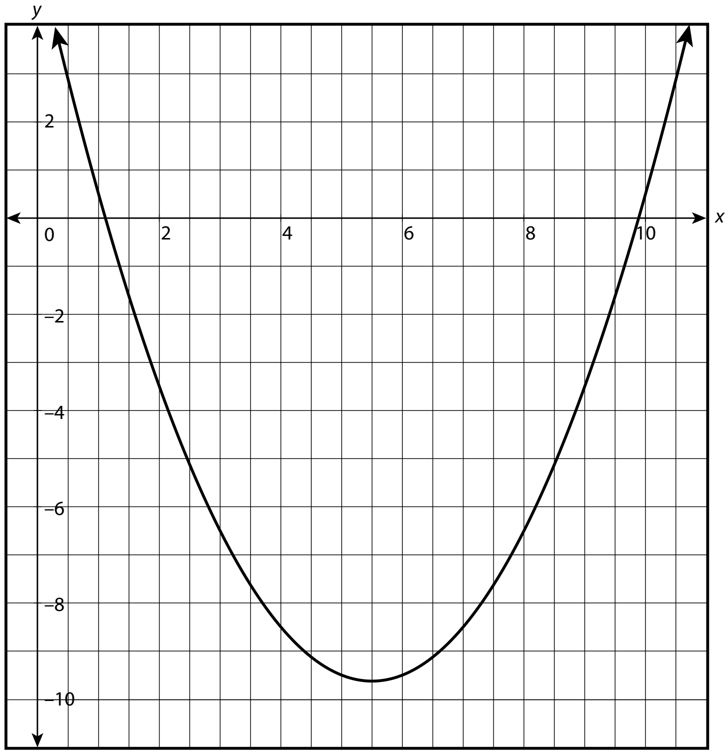
6.



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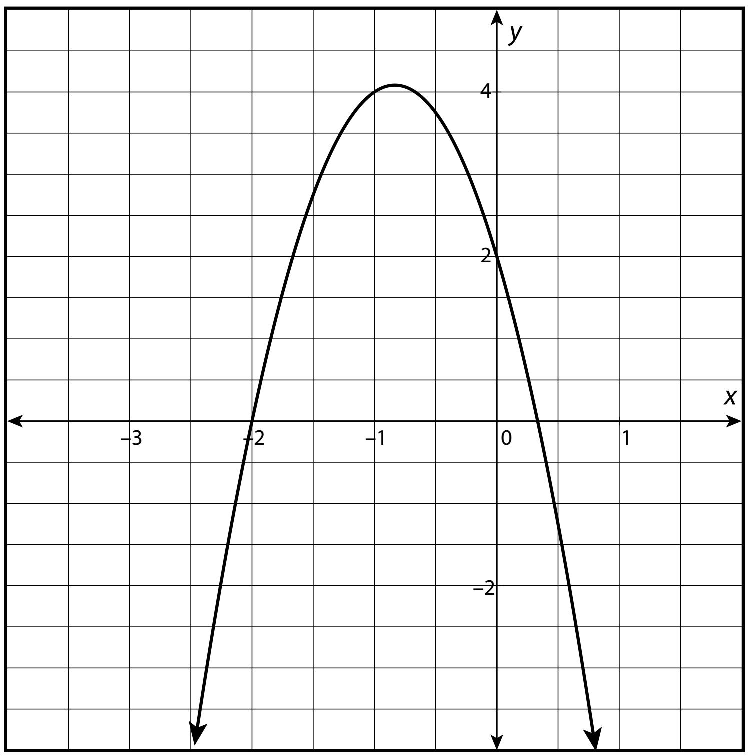

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Use the given information to solve the following problems.

8. A soccer ball is kicked from the ground and travels a parabolic path. The path can be modeled by the function $h(t) = -5t^2 + 19.5t$, where $h(t)$ is the height of the soccer ball in meters above the ground t seconds after being kicked. Assuming the ball lands on level ground, about how long is the ball in the air?

9. A golf ball is shot from the ground using a practice cannon and travels a parabolic path. The path of the ball can be modeled by the function $h(t) = -16t^2 + 150t$, where $h(t)$ is the height of the golf ball in meters above the ground t seconds after being shot. Assuming the ball lands on level ground, about how long does it take the golf ball to hit the ground?

10. The senior class is putting on a talent show to raise money for their senior trip. In the past, the profit from the talent show could be modeled by the function $P(x) = -16x^2 + 600x - 4000$, where x represents the ticket price in dollars. What are a reasonable domain and range for this function? For what domain value will the profits be maximized?

Practice 3.3.2: Identifying the Domain and Range of a Quadratic Function B

Use graphing technology to describe the domain and range of each quadratic function.

1. $y = 3x^2 - 4x + 2$

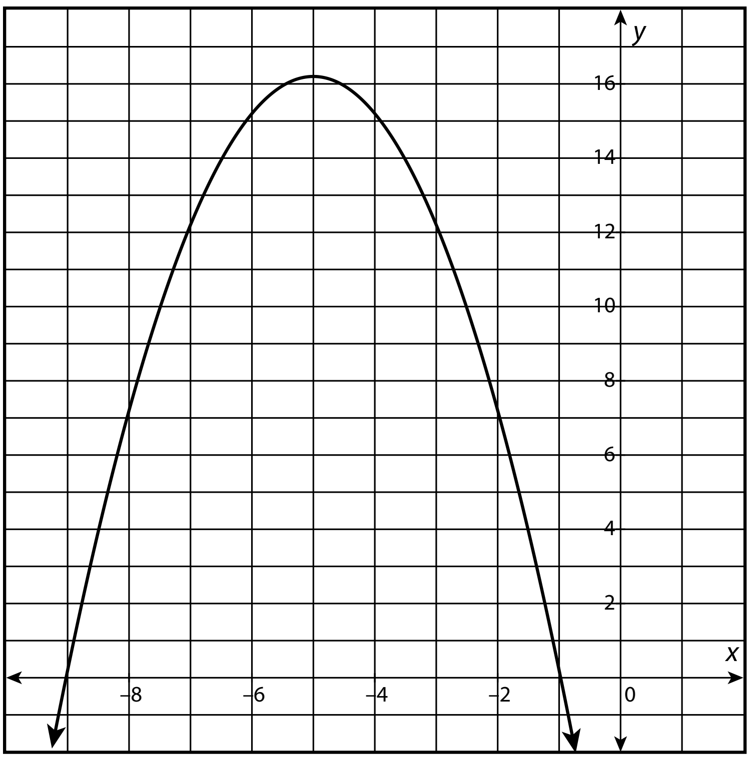
2. $y = -\frac{7}{4}x^2 - 32x - 5$

3. $f(x) = 6x^2 + 9x - 1$

4. $g(x) = 2x^2 - 12x - 9$

Describe the domain and range of each of the following functions in words and as an inequality.

5.



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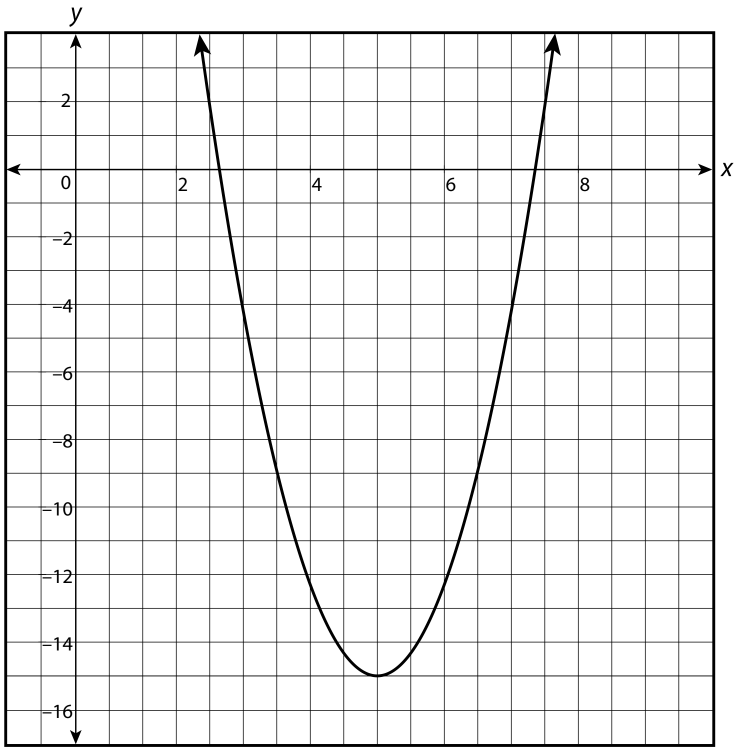
6.



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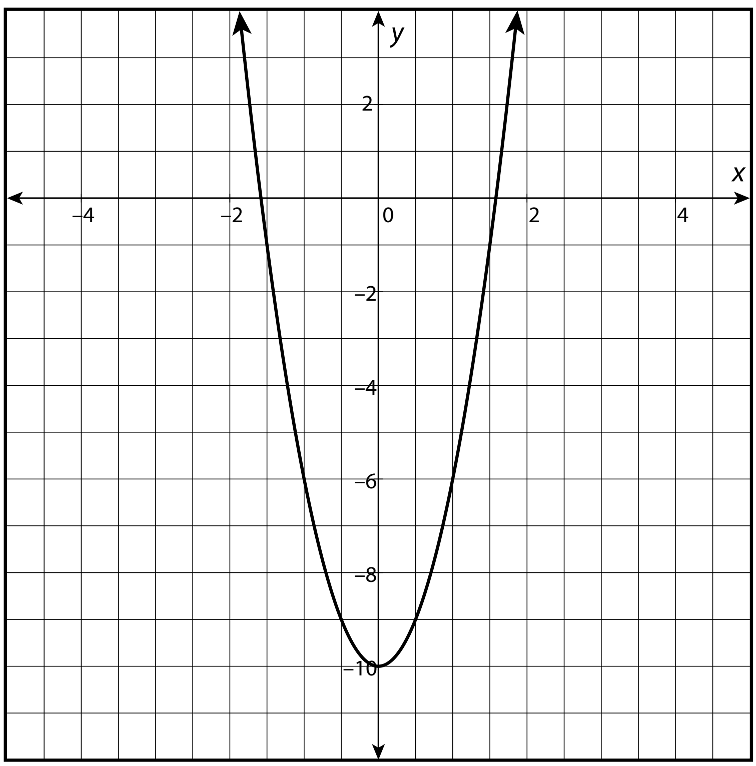

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Use the given information to solve the following problems.

8. A kickball is kicked from the ground and travels a parabolic path. The path can be modeled by the function $h(t) = -2t^2 + 20t$, where $h(t)$ is the height of the kickball in feet above the ground t seconds after being kicked. Assuming the ball lands on level ground, about how long is the ball in the air?

9. The height of baseballs thrown by an automatic baseball-pitching machine can be modeled by the function $h(t) = -16t^2 + 48t + 3.5$, where $h(t)$ is the height of the ball t seconds after being released. If the batter misses the ball, how long does it take the ball to hit the ground? Assume there is no net or catcher behind the plate to stop the ball.

10. A movie theater manager believes that the theater loses money as ticket prices go up. The theater's average weekly sales can be modeled by the quadratic function $R(x) = -700x^2 + 7700x + 245{,}000$, where $R(x)$ is the weekly revenue in dollars and x is the number of $0.50 increases in price. For what number of $0.50 increases will the theater continue to produce revenue? After how many $0.50 increases will the theater receive the greatest revenue?

Notes

Notes

Lesson 3.3.3: Identifying the Average Rate of Change

Warm-Up 3.3.3

Data on a certain car shows that its gas mileage can be modeled by a linear function. The car yields 45 miles per gallon (mpg) when driven at 40 miles per hour (mph) and 25 mpg when driven at 80 mph.

1. What is the rate of change of this car's gas mileage?

2. Create the linear model of the car's gas mileage as a function of its speed.

3. What does the rate of change in this model tell you about the car's gas mileage? Explain.

Scaffolded Practice 3.3.3

Example 1

Calculate the average rate of change for the function $f(x) = x^2 + 6x + 9$ between $x = 1$ and $x = 3$.

1. Evaluate the function for $x = 3$.

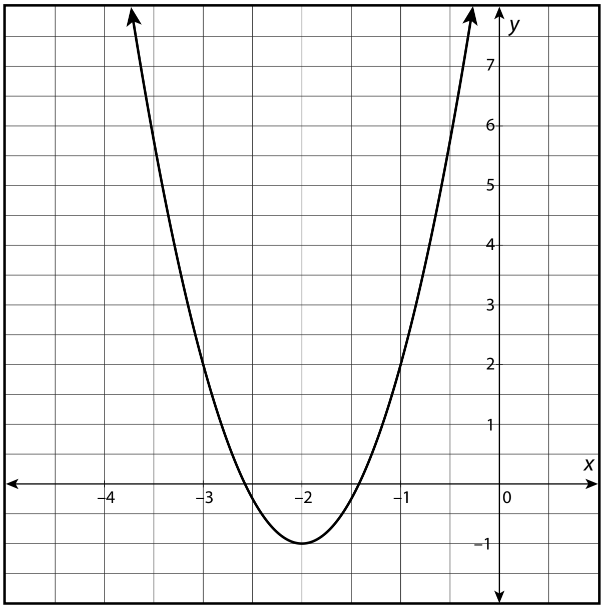
2. Evaluate the function for $x = 1$.

3. Use the average rate of change formula to determine the average rate of change between $x = 1$ and $x = 3$.

continued

Example 2

Use the graph of the function to calculate the average rate of change between $x = -3$ and $x = -2$.



Example 3

For the function $g(x) = (x - 3)^2 - 2$, is the average rate of change greater between $x = -1$ and $x = 0$ or between $x = 1$ and $x = 2$?

Example 4

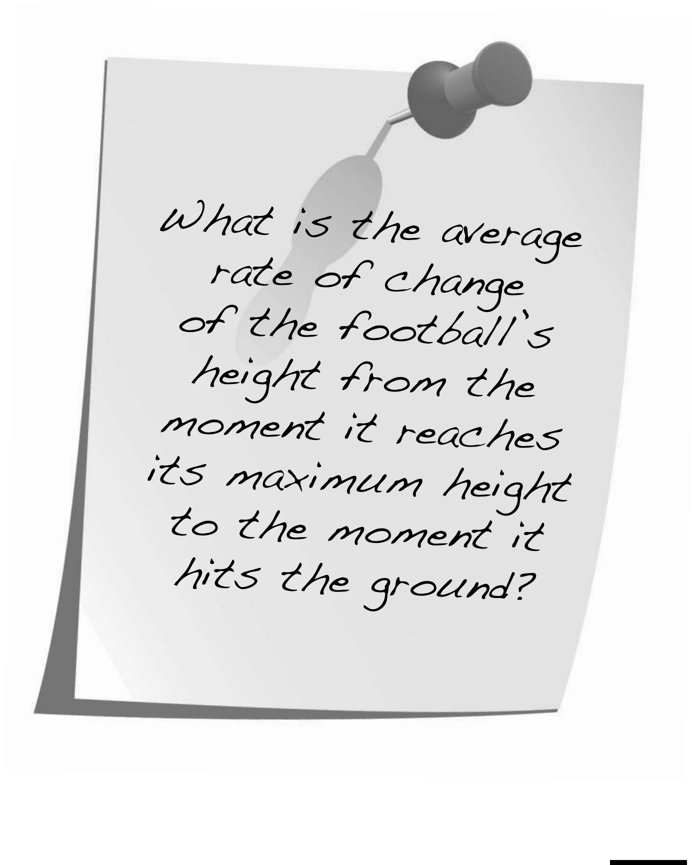
Find the average rate of change between $x = -0.75$ and $x = -0.25$ for the following function.

x	$g(x)$
–1	0
–0.75	3.44
–0.5	6.25
–0.25	8.44
0	10
0.25	10.94

Problem-Based Task 3.3.3: Is the Maximum High Enough?

SMP	
1 ✓	2 ✓
3	4
5	6 ✓
7	8

It is Super Bowl season and teams that have made the play-offs have specialists evaluating every aspect of their field game. One particular team received news that their recently injured kicker's field goal kick is modeled by the function $h(x) = -16(x - 1)^2 + 16$, where $h(x)$ is the height of the ball in feet x seconds after it is kicked. If the football needs to clear a 10-foot goalpost, will the ball make it over if this particular team member kicks it? What is the average rate of change of the football's height from the moment it reaches its maximum height to the moment it hits the ground?



Practice 3.3.3: Identifying the Average Rate of Change A

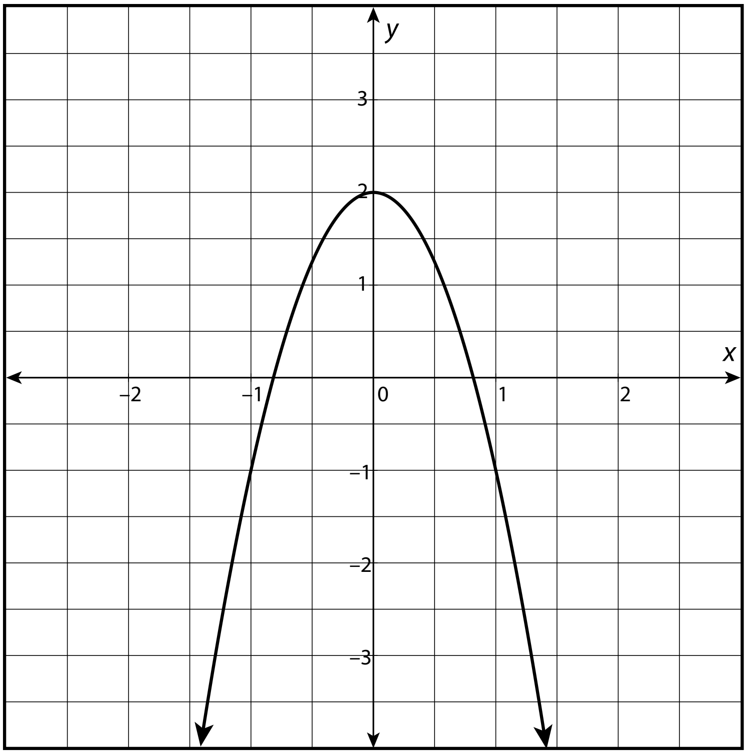
For problems 1–6, calculate the average rate of change of each function between $x = -1$ and $x = 1$.

1. $f(x) = 2(x + 1)^2 - 3$

2. $g(x) = 4 - 3(x - 1)^2$

3. $h(x) = x^2 - 4x + 6$

4.

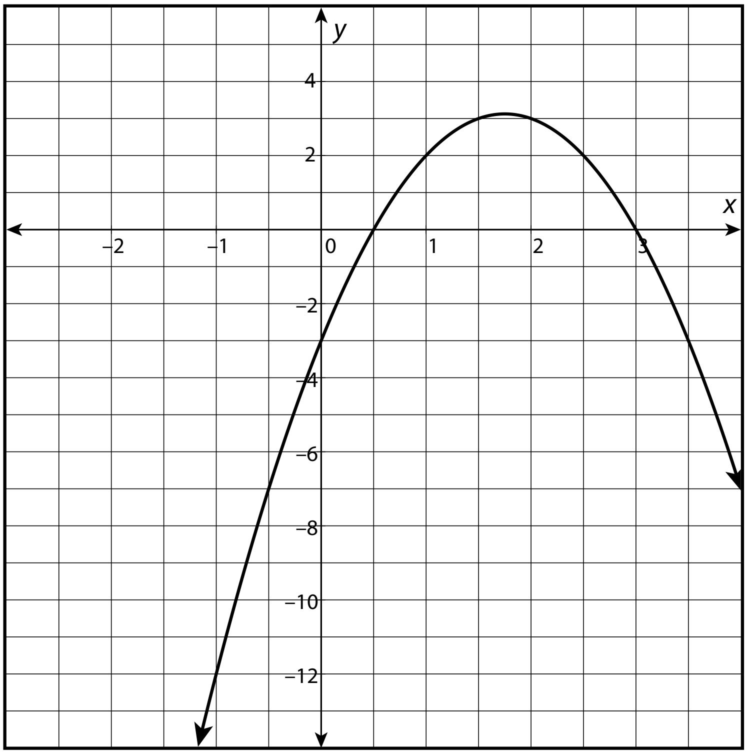


5.

x	y
–2	–1
–1.5	–1.75
–1	–4
–0.5	–7.75
0	–13
0.5	–19.75
1	–28
1.5	–37.75

continued

6.



For problems 7–9, determine whether the average rate of change is greater between $x = -2$ and $x = 0$ or between $x = 0$ and $x = 2$.

7. $y=\frac{1}{2}(x+2)^2-3$

8. $a(x) = -x^2 + 8x + 3$

9. $f(x) = 5x^2 - 6x + 4$

Read the scenario and use the information in it to answer the question.

10. A drop of rain falls from a height of 1,400 feet above the ground. The function $h(t) = -16t^2 + 1400$ is used to model the raindrop's height, $h(t)$, in feet t seconds after it starts to fall. What is the raindrop's average rate of change between 2 seconds and 3 seconds after it falls?

Practice 3.3.3: Identifying the Average Rate of Change B

For problems 1–6, calculate the average rate of change of each function between $x = -4$ and $x = -2$.

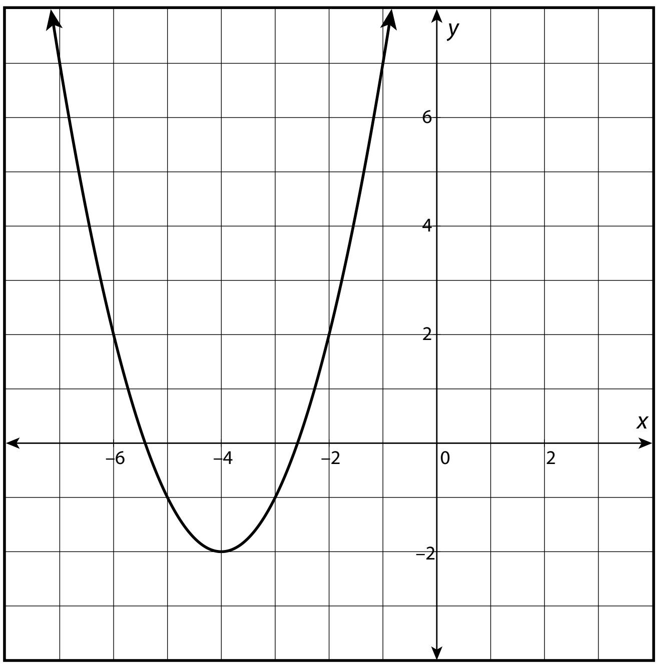
1. $f(x) = 2(x - 1)^2 - 4$
2. $g(x) = 12 - 2(x + 1)^2$
3. $h(x) = \frac{1}{4}x^2 - 1$
4.

x	$f(x)$
–6	110
–5	77
–4	50
–3	29
–2	14
–1	5
0	5
1	5

5.

x	$g(x)$
–6	162
–5	116
–4	78
–3	48
–2	26
–1	12
0	6
1	8

6.



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For problems 7–9, determine whether the average rate of change is greater between $x = -2$ and $x = -1$ or between $x = -1$ and $x = 0$.

7. $y=\frac{1}{2}x^2-x-8$

8. $a(x) = -2x^2 - 2x - 5$

9. $h(x) = 6x^2 + 31x - 12$

Read the scenario and use the information in it to answer the question.

10. A mother drops an apartment key down to her son from several floors above. The function $h(t) = -16t^2 + 60$ is used to model the key's height, $h(t)$, in feet t seconds after being released. What is the key's average rate of change between 1.5 seconds and 2 seconds after being dropped?

Name: Date:

Notes

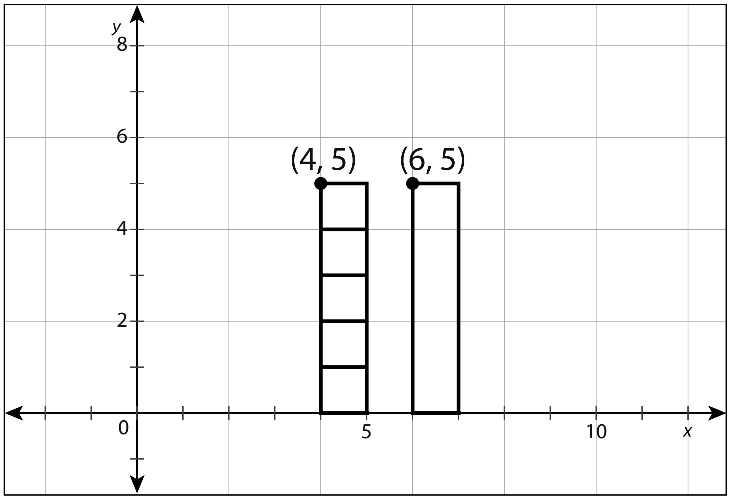
Name: **Date:**

Notes

Lesson 3.3.4: Writing Equivalent Forms of Quadratic Functions

Warm-Up 3.3.4

Joe is trying to use a hose to spray water over a stack of moving boxes and into a bird bath. The graph represents a stack of boxes on the left that the water must clear and a birdbath on the right that the water must fill. The water will follow a parabolic path.



1. One possible path the water could travel is given by $y=-\frac{1}{5}x^2+\frac{8}{5}x+\frac{9}{5}$, where y represents the height in feet and x represents the horizontal distance traveled in feet. What is the vertex of this parabola?

2. Determine the second x-intercept if one x-intercept of the path of the water is –1.

3. What is the maximum value of the quadratic function?

4. Sketch the graph of the path of the water.

5. Based on the graph, will the water clear the boxes? If it clears the boxes, will the water fill the birdbath?

Scaffolded Practice 3.3.4

Example 1

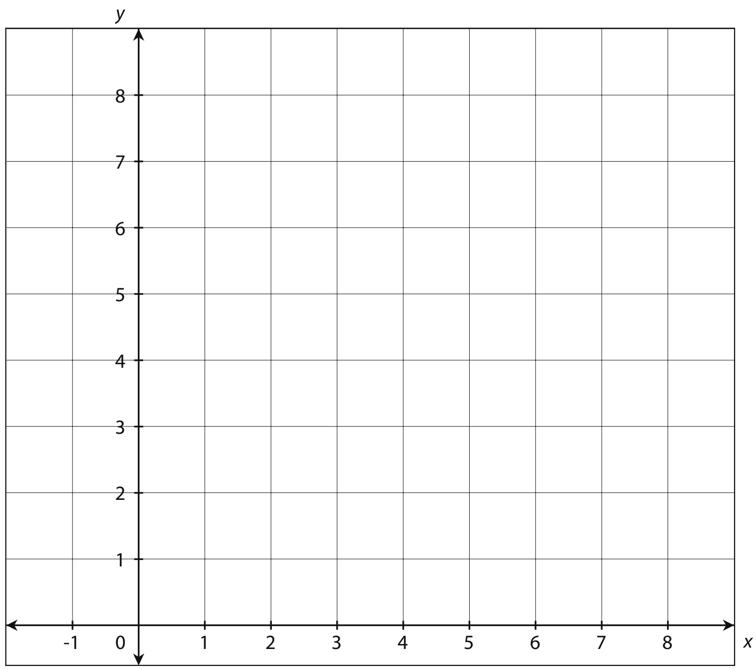
Suppose that the flight of a launched bottle rocket can be modeled by the function $f(x) = -(x - 1)(x - 6)$, where $f(x)$ measures the height above the ground in meters and x represents the horizontal distance in meters from the launching spot at the point (1, 0). How far does the bottle rocket travel in the horizontal direction from launch to landing? What is the maximum height the bottle rocket reaches? How far has the bottle rocket traveled horizontally when it reaches its maximum height? Graph the function.

1. Identify the x-intercepts of the function.

2. Determine the maximum height of the bottle rocket.

3. Determine the horizontal distance from the launch point to the maximum height of the bottle rocket.

4. Graph the function.



continued

Example 2

Reducing the cost of an item can result in a greater number of sales. The revenue function that predicts the revenue in dollars, $R(x)$, for each \$1 change in price, x, for a particular item is $R(x) = -100(x - 7)^2 + 28{,}900$. What is the maximum value of the function? What does the maximum value mean in the context of the problem? What price increase maximizes the revenue and what does it mean in the context of the problem? Graph the function.

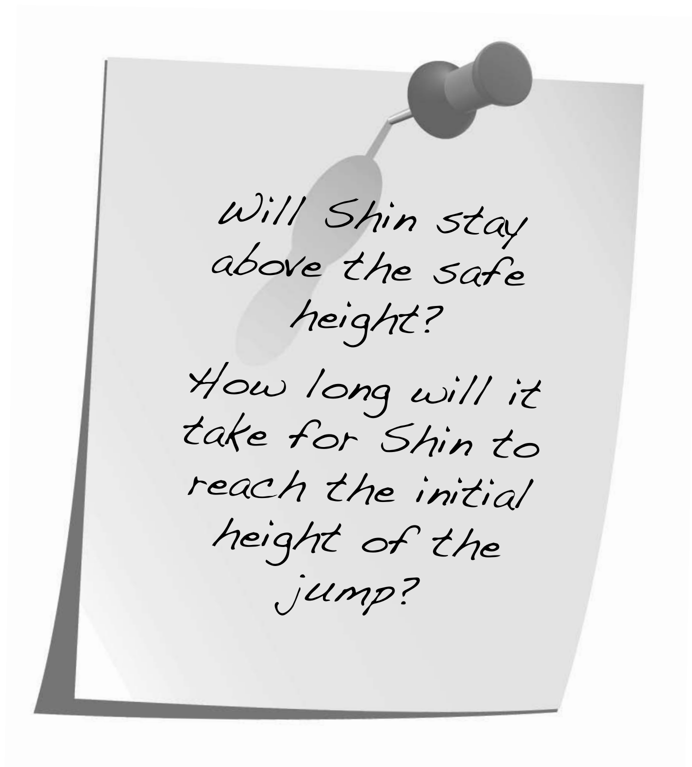
Example 3

A football is kicked and follows a path given by $f(x) = -0.03x^2 + 1.8x$, where $f(x)$ represents the height of the ball in feet and x represents the horizontal distance in feet. What is the maximum height the ball reaches? What horizontal distance maximizes the height? Graph the function.

Problem-Based Task 3.3.4: Is the Glider Safe?

SMP	
1 ✓	2
3 ✓	4 ✓
5	6
7	8

Shin is a beginner hang glider. He's practicing jumping from a certain height, dipping initially, and then rising. Shin should dip to a height no lower than 6 feet above the ground, which is considered a safe height, before changing direction and beginning to rise. The position of Shin's hang glider is given by $y = (x - 4)(x - 6)$, with x representing the time in seconds since Shin starts the initial jump and y representing the distance in feet from the safe height. Will Shin stay above the safe height? How long will it take for Shin to reach the initial height of the jump?



Practice 3.3.4: Writing Equivalent Forms of Quadratic Functions A

Use the given functions to complete all parts of problems 1–3.

1. $f(x) = x^2 - 6x + 8$
   a. Identify the y-intercept.
   b. Identify the vertex.
   c. Identify whether the function has a maximum or minimum.

2. $f(x) = -0.5(x + 2)(x - 4)$
   a. Identify the x-intercepts.
   b. Identify the y-intercept.
   c. Identify the axis of symmetry.
   d. Identify the vertex.

3. $f(x) = -16(x - 1)^2 + 10$
   a. Identify the vertex.
   b. Identify whether the function has a maximum or minimum.

Use the given information in each scenario that follows to complete the remaining problems.

4. A bird is descending toward a lake to catch a fish. The bird's flight can be modeled by the equation $h(t) = t^2 - 14t + 40$, where $h(t)$ is the bird's height above the water in feet and t is the time in seconds since you saw the bird. Graph the function. What is the vertex? What does the minimum value mean in the context of the problem?

5. A military pilot fires a test missile whose path can be modeled by the equation $f(x) = -(x - 40)(x + 2)$, where $f(x)$ is the height of the missile in miles and x is the number of seconds since the missile was fired. Graph this function. What are the x-intercepts and what do they mean in the context of the problem? After how many seconds is the height of the missile the same as the initial height?

continued

6. The path of a snowboarder performing stunts is given by the equation $f(x) = -16(x - 2)(x + 1)$, where x is time in seconds and $f(x)$ is the height of the snowboarder above the ground. Graph the function. What are the x-intercepts? Explain the meaning of the x-intercepts in the context of the problem. How long does the stunt last?

7. The flight of a paper airplane follows the quadratic equation $H(x) = -(x - 3)^2 + 25$, where $H(x)$ represents the height of the paper airplane in feet, and x is the horizontal distance in feet the airplane travels after it is thrown. Graph the function. What is the vertex? Explain the meaning of the vertex in the context of the problem.

8. The height of a golfer's ball is given by the equation $y = -16x^2 + 32x$, where y represents the height in feet and x represents the time in seconds. Graph the function. What is the vertex and what does it mean in the context of the problem?

9. The revenue, $R(x)$, generated by an increase in price of x dollars for an item is represented by the equation $R(x) = -5(x - 15)(x + 5)$. Graph the function. What are the x-intercepts and what do they represent in the context of the problem? What value of x maximizes the revenue?

10. Reducing the cost of an item can result in a greater number of sales. The revenue function that predicts the revenue in dollars, $R(x)$, for each \$1 decrease in price, x, is $R(x) = -(x - 7)^2 + 289$. Graph the function. What is the vertex and what does it represent in the context of the problem?

Practice 3.3.4: Writing Equivalent Forms of Quadratic Functions B

Use the given functions to complete all parts of problems 1–3.

1. $f(x) = x^2 - 8x + 12$
   a. Identify the y-intercept.
   b. Identify the vertex.
   c. Identify whether the function has a maximum or minimum.

2. $f(x) = -2(x - 3)(x + 5)$
   a. Identify the x-intercepts.
   b. Identify the y-intercept.
   c. Identify the axis of symmetry.
   d. Identify the vertex.

3. $f(x) = -16(x - 3)^2$
   a. Identify the vertex.
   b. Identify whether the function has a maximum or minimum.

Use the given information in each scenario that follows to complete the remaining problems.

4. A butterfly descends toward the ground and then flies back up. The butterfly's descent can be modeled by the equation $h(t) = t^2 - 10t + 26$, where $h(t)$ is the butterfly's height above the ground in feet and t is the time in seconds since you saw the butterfly. Graph the function and identify the vertex. What is the meaning of the vertex in the context of the problem?

5. A cliff diver jumps upward from the edge of a cliff then begins to descend, so that his path follows a parabola. The diver's height, $h(t)$, above the water in feet is given by $h(t) = -2(t - 1)^2 + 52$, where t represents the time in seconds. Graph the function. What is the vertex and what does it represent in the context of the problem? How many seconds after the start of the dive does the diver reach the initial height?

continued

6. The revenue of producing and selling widgets is given by the function $R(x) = -8(x - 50)(x - 2)$, where x is the number of widgets produced and $R(x)$ is the amount of revenue in dollars. Graph the function. What are the x-intercepts and what do they represent in the context of the problem? What number of widgets maximizes the revenue?

7. A football is kicked and follows a path given by $y = -0.03x^2 + 1.2x$, where y represents the height of the ball in feet and x represents the horizontal distance in feet. Graph the function. What is the vertex and what does it mean in the context of the problem? How far does the ball travel in the horizontal direction?

8. A frog hops from the bank of a creek onto a lily pad. The path of the jump can be modeled by the equation $h(x) = -\frac{1}{2}(x-2)^2 + 4$, where $h(x)$ is the frog's height in feet above the water and x is the number of seconds since the frog jumped. Graph the function. What does the vertex represent in the context of the problem? What is the axis of symmetry? After how many seconds does the height of the frog reach the initial height?

9. The revenue, $R(x)$, generated by an increase in price of x dollars for an item is represented by the equation $R(x) = -2x^2 + 20x + 150$. Graph the function and identify the vertex. What does the vertex represent in the context of the problem? What is the axis of symmetry? What increase in price results in the same revenue as not increasing the price at all?

10. Reducing the cost of an item can result in a greater number of sales. The revenue function that predicts the revenue in dollars, $R(x)$, for each \$1 decrease in price, x, for a certain item is $R(x) = -(x - 26)(x + 10)$. Graph the function. Identify the x-intercepts. What do the x-intercepts represent in the context of the problem? What is the axis of symmetry? What increase in price results in the same revenue as not increasing the price at all?

Name: **Date:**

Notes

Name: Date:

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Lesson 3.4.1: Replacing $f(x)$ with $f(x) + k$ and $f(x + k)$

Warm-Up 3.4.1

As a fund-raiser for the senior prom, the student council has decided to charge a small fee for parking in the school parking lot. Students can purchase yearly parking passes for $5 each. The school spent a total of $10 to buy a year's supply of the tags the students hang in their windows to verify that they paid for the parking spot.

1. Build a function that models the profit the school will make selling parking passes.

2. Graph the function.

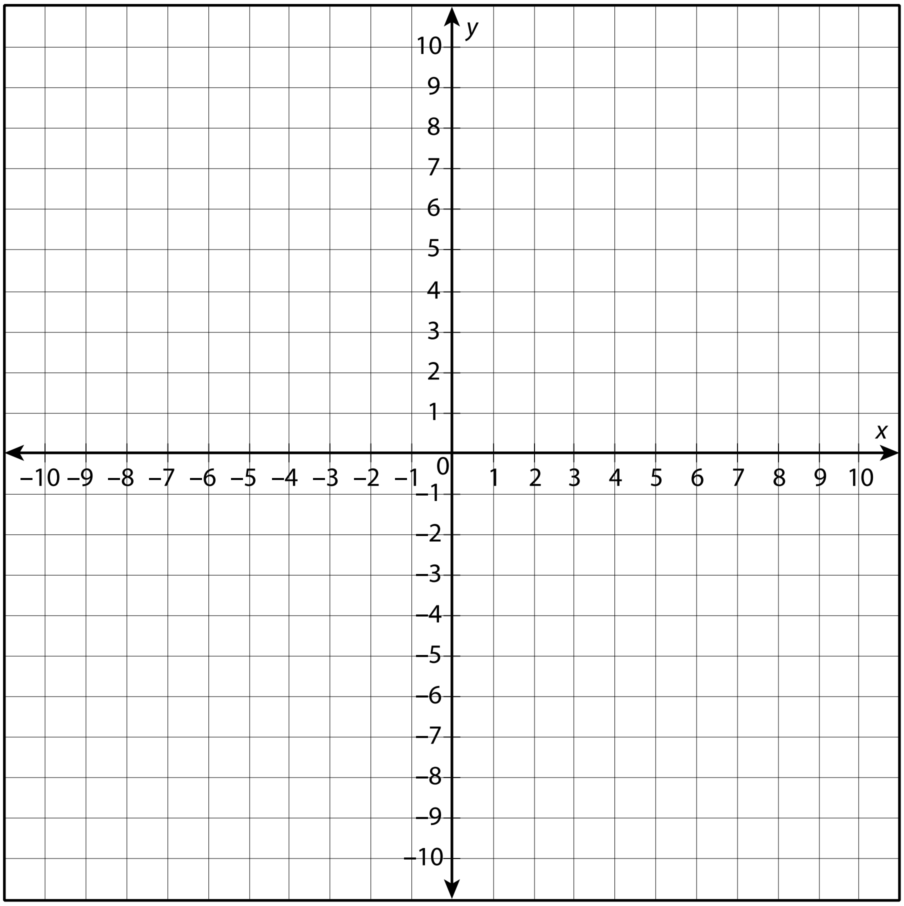
3. What would be the effect on the graph if the school were able to find a company that only charges $8 for the tags?

Scaffolded Practice 3.4.1

Example 1

Consider the function $f(x) = x^2$ and the constant $k = 2$. What is $f(x) + k$? How are the graphs of $f(x)$ and $f(x) + k$ different?

1. Substitute the value of k into the function.
2. Use a table of values to graph the functions on the same coordinate plane.



3. Compare the graphs of the functions.

continued

Example 2

Consider the function $f(x) = x^2$ and the constant $k = -3$. What is $f(x) + k$? How are the graphs of $f(x)$ and $f(x) + k$ different?

Example 3

Consider the function $f(x) = x^2$, its graph, and the constant $k = 4$. What is $f(x + k)$? How are the graphs of $f(x)$ and $f(x + k)$ different?

Example 4

Consider the function $f(x) = x^2$ and the constant $k = -1$. What is $f(x + k)$? How are the graphs of $f(x)$ and $f(x + k)$ different?

Example 5

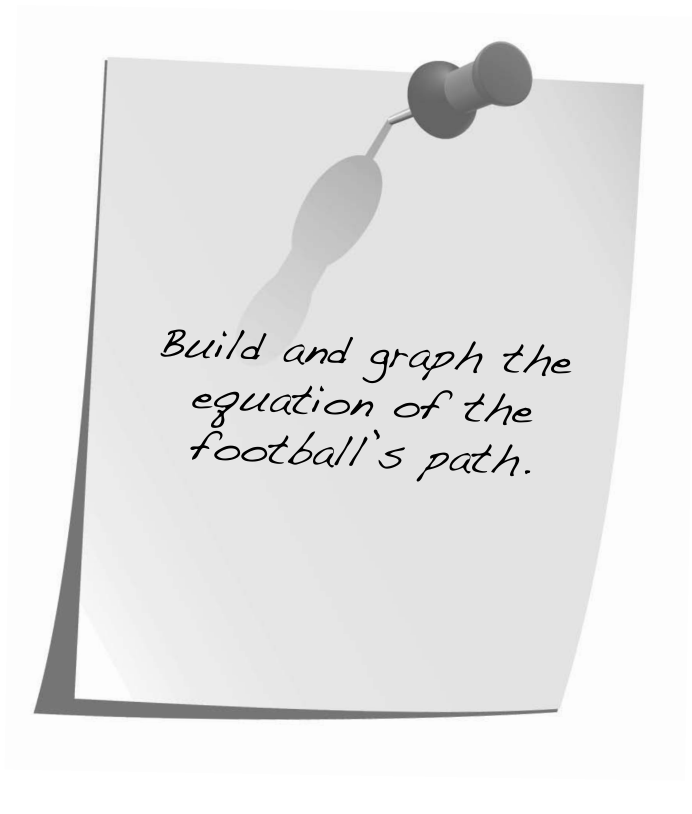
The revenue function for a model helicopter company is modeled by the curve $f(x) = -5x^2 + 400x$, where x is the number of helicopters built per month and $f(x)$ is the revenue. The owner wants to include rent in the revenue equation to determine the company's profit per month. The company pays \$2,250 per month to rent its warehouse. In terms of $f(x)$, what equation now describes the company's profit per month? Compare the vertices of the original function and the transformed function.

Problem-Based Task 3.4.1: The Catch

SMP	
1	2 ✓
3	4 ✓
5	6
7 ✓	8

On the last play of a football game, the offense is on the opposing team's 35-yard line. The offense is losing by 4 points, but can win by making a touchdown. The quarterback backs away 5 yards from behind the line of scrimmage and throws the ball to his receiver, who makes the catch at the goal line for the touchdown and the win. The quarterback's release point is 6 feet above the ground, the same height at which the receiver caught the ball. Also, the ball was thrown such that its maximum height was 15 feet above the ground.

Given this information, build and graph the equation of the football's path, with the *x*-axis representing the distance from the line of scrimmage and the *y*-axis as the height of the football above the ground, with distances measured in yards.



Practice 3.4.1: Replacing $f(x)$ with $f(x) + k$ and $f(x + k)$ A

For problems 1–3, let $f(x) = x^2$. Write a function that translates $f(x)$ as described.

1. 2 units to the left

2. 3 units up

3. 5 units to the right and 2 units down

For problems 4–6, let $f(x) = x^2$. Graph $g(x)$ by translating the graph of $f(x)$. State the vertex of the translated function.

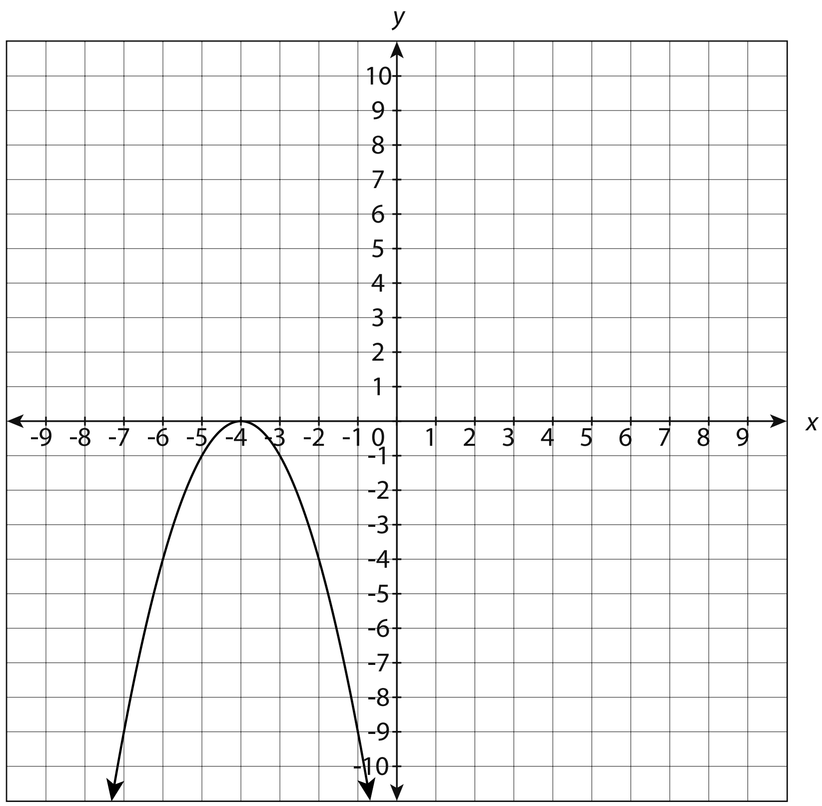
4. $g(x) = (x - 2)^2$

5. $g(x) = x^2 - 4$

6. $g(x) = (x - 5)^2 - 2$

Use what you know about translations of functions to solve each problem.

7. The following graph is a translation of $f(x) = -x^2$. Write an equation for the graph and state the value of k that was used to transform the function.



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8. A mother and her daughter went golfing. The mother hit first. Her ball followed the path modeled by the equation $f(x) = -0.0009x^2 + 0.2088x$ in the direction of the hole, and landed 18 yards short of the hole. The daughter teed off 18 yards closer to the hole because she is a beginner. She realized that if she could hit the ball on the same trajectory as her mother, her ball would land right by the hole. What is the equation that describes the path that the daughter's ball should follow?

9. A basketball is thrown from a height of 4 feet so that its path is modeled by the function $f(x) = -0.03x^2 + 1.3x + 4$. If the exact same shot is taken from a balcony that is 12 feet above where the original shooter was standing, how far away will the ball hit the ground? What is the equation that models this shot?

10. Simon has a toy that launches hollow plastic balls. The launched balls always follow a path modeled by the function $f(x) = -\frac{1}{8}(x-8)^2 + 8$ when the launcher is at the "origin." If the launcher is lifted up 2 feet and moved forward 5 feet, will a launched ball land in a basket that is on a 4-foot high stool 20 feet from the origin? What is the function that models this new launcher position?

Practice 3.4.1: Replacing $f(x)$ with $f(x) + k$ and $f(x + k)$ B

For problems 1–3, let $f(x) = x^2$. Write a function that translates $f(x)$ as described.

1. 3 units to the right

2. 4 units down

3. 6 units to the left and 1 unit down

For problems 4–6, let $f(x) = x^2$. Graph $g(x)$ by translating the graph of $f(x)$.

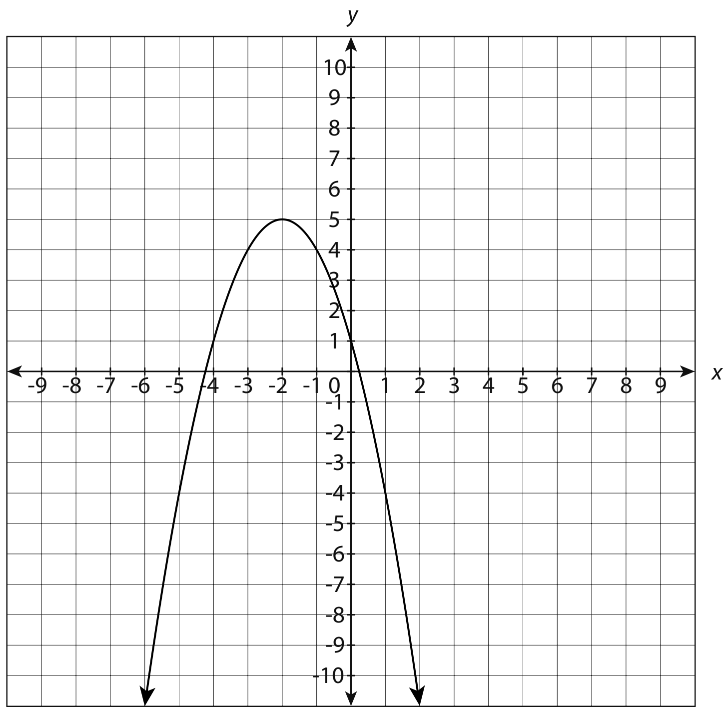
4. $g(x) = (x + 4)^2$

5. $g(x) = x^2 + 1$

6. $g(x) = (x - 2)^2 - 5$

Use what you know about translations of functions to solve each problem.

7. The following graph is a translation of $f(x) = -x^2$. Write an equation for the graph. State the value of k that was used to transform the function horizontally and the value of k used to transform the function vertically.



continued

8. A mother and her son went golfing. The mother hit first. Her ball followed the path modeled by the equation $f(x) = -0.0008x^2 + 0.24x$ in the direction of the hole, and landed 12 yards short of the hole. The son teed off 12 yards closer to the hole because he is a beginner. He realized that if he could hit the ball on the same trajectory as his mother, his ball would land right by the hole. What is the equation that describes the path that the son's ball should follow?

9. A paper wad is thrown from a height of 4 feet so that its path is modeled by the function $f(x) = -0.05x^2 + x + 4$. If the exact same shot is taken from a balcony that is 15 feet above where the original shooter was standing, how far away will the paper wad hit the ground? What is the equation that models this shot?

10. Suzanne has a toy that launches rubber bands. The rubber bands always follow a path modeled by the function $f(x) = -0.4(x - 5)^2 + 10$ when the launcher is at the "origin." If the launcher is lifted up 4 feet and moved forward 4 feet, will a launched rubber band hit a painted target on a 10-foot-tall tree branch that is 12 feet from the origin? What is the function that models this new launcher position?

Notes

Name: Date:

Notes

Lesson 3.4.2: Replacing $f(x)$ with $k \bullet f(x)$ and $f(k \bullet x)$

Warm-Up 3.4.2

An architectural firm is designing a one-story, square office building that is required to have a 4-foot space in every exterior wall for an exit. The area of the office building is given by the equation $f(x) = (x + 4)(x + 4) = (x + 4)^2$.

1. Given $f(x) = (x + 4)^2$, what would be the area of the building if $x = 20$ feet?

2. If the firm used the same blueprint with the area found in problem 1 to design a building with 3 identical floors, what would be the total area of the new building's office space?

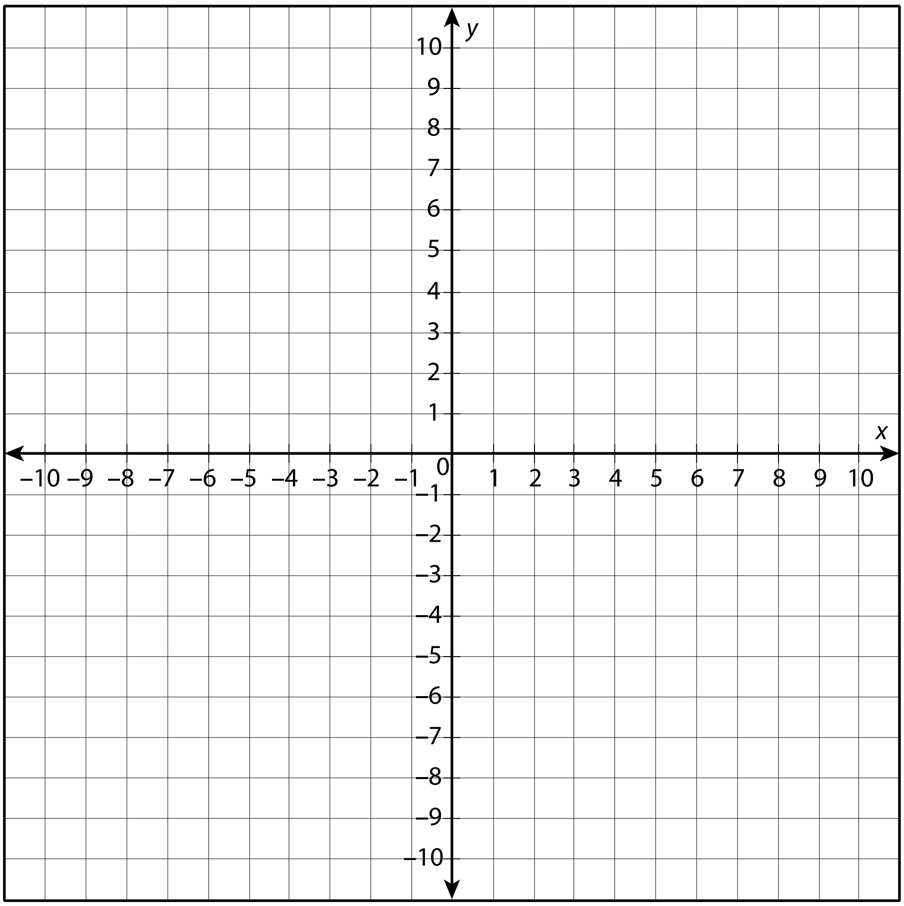
3. If the firm were to double the length of each wall in the original design, what would be the new area of the one-story building?

Scaffolded Practice 3.4.2

Example 1

Consider the function $f(x) = x^2$, its graph, and the constant $k = 2$. What is $k \bullet f(x)$? How are the graphs of $f(x)$ and $k \bullet f(x)$ different? How are they the same?

1. Substitute the value of k into the function.
2. Use a table of values to graph the functions.



3. Compare the graphs.

continued

Example 2

Consider the function $f(x) = x^2 - 81$, its graph, and the constant $k = 3$. What is $f(k \bullet x)$? How do the vertices and intercepts of $f(x)$ and $f(k \bullet x)$ compare?

Example 3

Consider the function $f(x) = x^2 - 6x + 8$, its graph, and the constant $k = -1$. What is $k \bullet f(x)$? How do the graphs of $f(x)$ and $k \bullet f(x)$ compare?

Example 4

Consider the function $f(x) = x^2 - 6x + 8$, its graph, and the constant $k = -1$. What is $f(k \bullet x)$? How do the graphs of $f(x)$ and $f(k \bullet x)$ compare?

Example 5

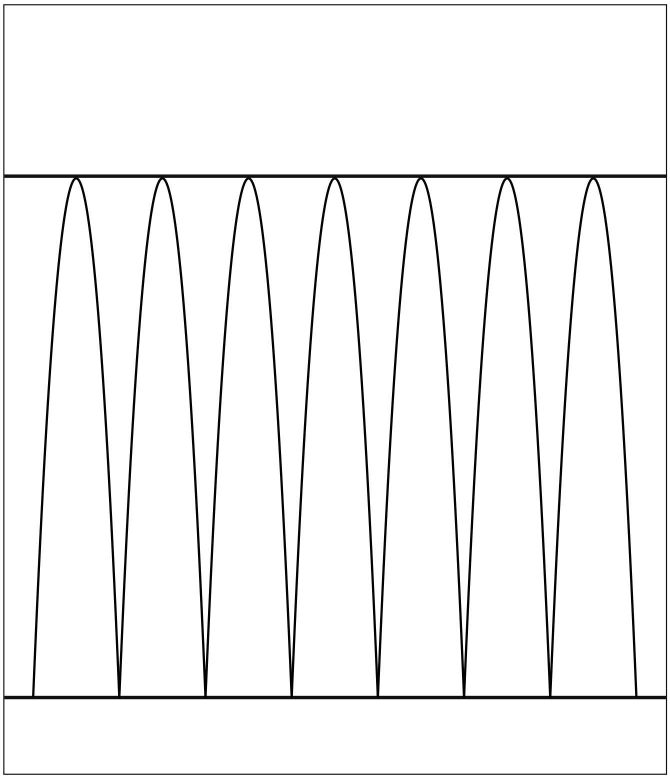
The dimensions of a rectangular garden edged with wood are such that the longer sides are 3 times the length of the shorter sides. Keeping the same ratio of side lengths, which would result in having a larger garden area: making the existing sides 5 times longer, or building 4 more gardens that are equal in size to the existing garden?

Problem-Based Task 3.4.2: Fewer Parabolas, Please

SMP	
1 ✓	2
3	4 ✓
5	6
7	8 ✓

A city hired a civil engineering firm to draw up plans for a 420-foot bridge that would use 7 downward-facing parabolic arches to support the span. The resulting plans called for the arches to be 75 feet high, with a distance of 60 feet between the bases of each arch, as shown in the following diagram.

After seeing the drawings, city councilors asked the civil engineering firm to make a second set of drawings using only 5 evenly spaced parabolic curves but covering the same 420-foot span. What are the equations of the 5 parabolas in the new plan? Let the *y*-axis represent the axis of symmetry for the left-most parabola, and let the *x*-axis represent the bottom of the arches.



What are the equations of the 5 parabolas in the new plan?

Practice 3.4.2: Replacing *f*(*x*) with *k* • *f*(*x*) and *f*(*k* • *x*) A

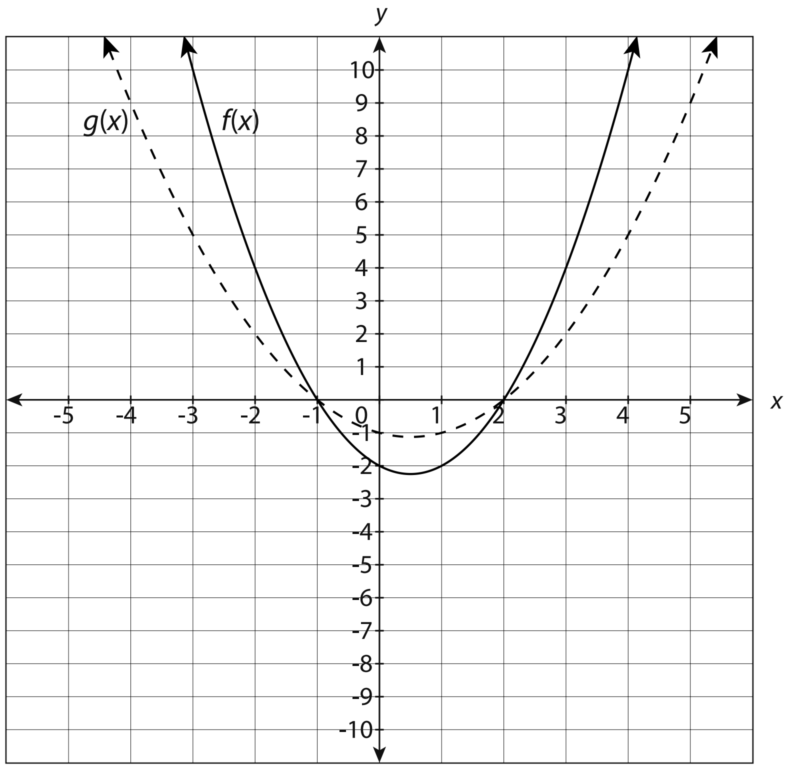
Use what you have learned about transformations of functions to solve problems 1 and 2.

1. For the function $f(x) = x^2 + x - 6$, find $2 \bullet f(x)$, and describe the changes that occur to the graph of $f(x)$ as a result of multiplying the function by 2. Check your answers by comparing the two functions on your graphing calculator.

2. For the function $f(x) = x^2 + x$, find $f(3x)$, and describe the changes that occur to the graph of $f(x)$ as a result of multiplying the variable x by 3. Check your answers by comparing the two functions on your graphing calculator.

Use the graphs and the given information to complete problems 3 and 4.

3. Consider the graphs of the functions $f(x)$ and $g(x)$. The equation for $f(x)$ is $f(x) = x^2 - x - 2$. What could be the equation for $g(x)$?



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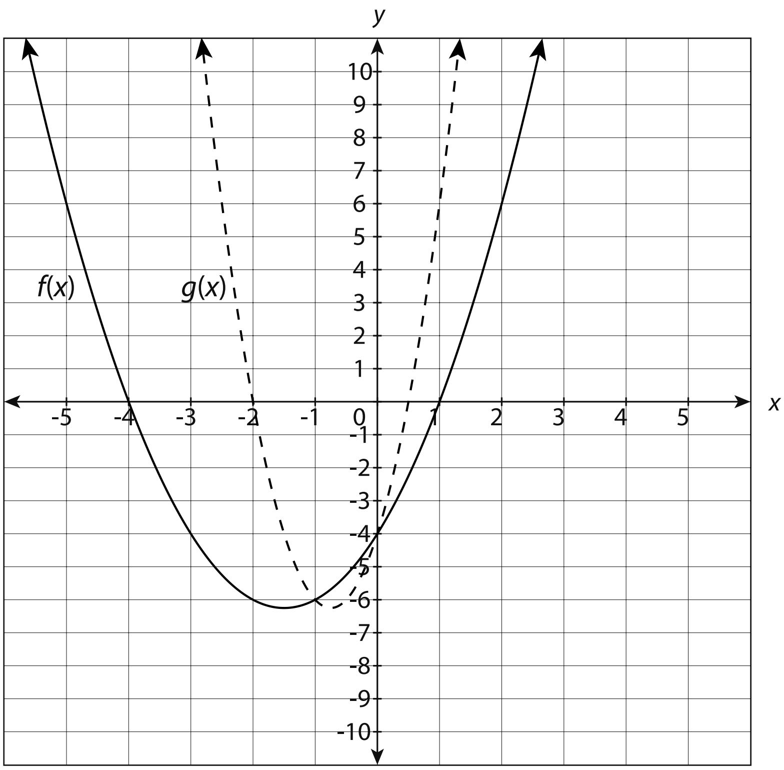
4. Consider the graphs of the functions $f(x)$ and $g(x)$. The equation for $f(x)$ is $f(x) = x^2 + 3x - 4$. What could be the equation for $g(x)$?



Complete each of the following tasks for the functions in problems 5–7.

- Graph $f(x)$ and $g(x)$ on your graphing calculator.
- Determine the scale factor and the transformation(s): horizontal stretch, horizontal compression, vertical stretch, vertical compression, reflection over the x-axis, or reflection over the y-axis.
- Describe the similarities and differences of the graphs.

5. $f(x) = x^2 - x - 2$; $g(x) = -2f(x)$

6. $f(x) = x^2 - x - 2$; $g(x) = f(-2x)$

7. $f(x) = x^2 - 1$; $g(x) = -\frac{1}{2} \bullet f(x)$

continued

Read each scenario and use the given information to solve problems 8–10.

8. A farmer has a rectangular goat pen such that one side is 2 times as long as the other side. He would like to have more space for his goats, and he is deciding between two options. He could either double the lengths of the sides of the existing pen, or he could build a second pen of the same size as the first. Which option would give him the most area for his goats? Explain your answer in terms of $k \bullet f(x)$ and $f(k \bullet x)$.

9. A company that produces skateboards knows the equation that models profit per month is $f(x) = 3x^2 + 300x$, where x is the price charged per skateboard. If the company plans to expand with the hopes of doubling its profits, should the new model for the company's profit be $f(2x)$, $f\left(\frac{1}{2}x\right)$, $2 \bullet f(x)$, or $\frac{1}{2} \bullet f(x)$? Explain.

10. Jada and Jayla are twins on the same softball team. They can each hit the ball so that it follows a path modeled by the equation $f(x) = -0.01x^2 + 0.98x + 2$. Jada says that the ball would go farther if it followed the path $g(x) = f(2x)$. Jayla says the ball would go farther if it followed the path $g(x) = 2 \bullet f(x)$. Who is correct? Which equation for $g(x)$ would allow the ball to achieve the same height as the ball in the original equation?

Practice 3.4.2: Replacing $f(x)$ with $k \bullet f(x)$ and $f(k \bullet x)$ B

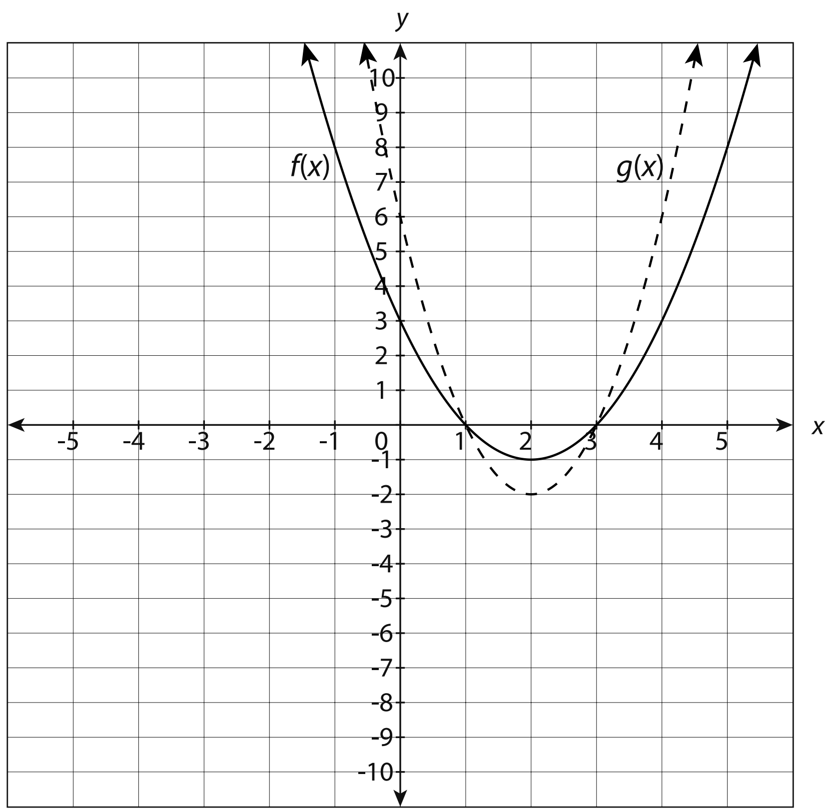
Use what you have learned about transformations of functions to solve problems 1 and 2.

1. For the function $f(x) = x^2 - 3x - 4$, find $3 \bullet f(x)$, and describe the changes that occur to the graph of $f(x)$ as a result of multiplying the function by 3. Check your answers by comparing the two functions on your graphing calculator.

2. For the function $f(x) = x^2 - 3x + 2$, find $f(2x)$, and describe the changes that occur to the graph of $f(x)$ as a result of multiplying the variable x by 2. Check your answers by comparing the two functions on your graphing calculator.

Use the graphs and the given information to complete problems 3 and 4.

3. Consider the graphs of the functions $f(x)$ and $g(x)$. The equation for $f(x)$ is $f(x) = x^2 - 4x + 3$. What could be the equation for $g(x)$?



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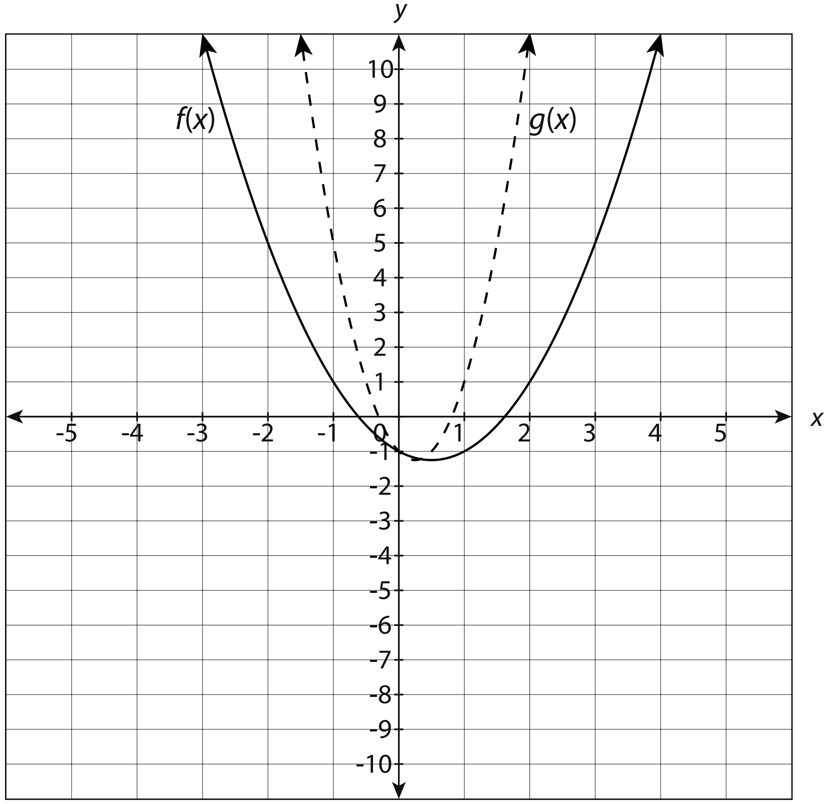
4. Consider the graphs of the functions $f(x)$ and $g(x)$. The equation for $f(x)$ is $f(x) = x^2 - x - 1$. What could be the equation for $g(x)$?



Complete each of the following tasks for the functions in problems 5–7.

- Graph $f(x)$ and $g(x)$ on your graphing calculator.
- Determine the scale factor and the transformation(s): horizontal stretch, horizontal compression, vertical stretch, vertical compression, reflection over the x-axis, or reflection over the y-axis.
- Describe the similarities and differences of the graphs.

5. $f(x) = x^2 + 2x - 3$; $g(x) = -2f(x)$

6. $f(x) = x^2 + 2x - 3$; $g(x) = f(-2x)$

7. $f(x) = x^2 - 4$; $g(x) = -\frac{1}{2} \bullet f(x)$

continued

Read each scenario and use the given information to solve problems 8–10.

8. A celebrity has a rectangular closet such that one side is 3 times as long as the other side. He would like to have more space for his coats, and he is deciding between two options. He could either triple the lengths of the sides of the existing closet, or he could build 2 more closets of the same size. Which option would give him the most area for his coats? Explain.

9. Dina manages a swimwear store in a beach town. She knows the equation that models the store's profit per month in the summer is $f(x) = 3x^2 + 300x$, where x is the average price charged per swimsuit. If swimsuit sales drop by half in the winter, is the new model for the store's profit $f(2x)$, $f\left(\frac{1}{2}x\right)$, $2 \bullet f(x)$, or $\frac{1}{2}f(x)$? Explain.

10. Zion and Zavier built a small catapult that launches beanbags for physics class. The catapult can launch a beanbag so that the bag follows a path modeled by the equation $f(x) = -0.004x^2 + 0.792x + 1.6$. Zion says that the beanbag would go farther if it followed the path $g(x) = f\left(\frac{1}{2}x\right)$. Zavier says the beanbag would go farther if it followed the path $g(x) = 2 \bullet f(x)$. Who is correct? Which equation for $g(x)$ would allow a launched beanbag to achieve the same height as a beanbag in the original equation? Which equation for $g(x)$ would allow a launched beanbag to go the same distance as a beanbag in the original equation?

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Lesson 3.5.1: Building Quadratic Functions From Context

Warm-Up 3.5.1

The height of an object launched into the air, also known as a projectile, depends on the height from which it was launched, the speed with which it was launched, and the force of gravity. The height of a projectile after t seconds is given by the function $f(t) = h + vt - \frac{1}{2}gt^2$, where h represents the starting height, v represents the vertical velocity, and g represents acceleration due to gravity. *Note*: On Earth, $g \approx 32$ ft/s^2.

1. A cannon on top of a cliff fires a cannonball over the ocean. The cannonball is launched from 192 feet above sea level with a vertical velocity of 176 ft/s. How long is the cannonball in the air before it hits the ocean?

2. A second cannon on a higher cliff launches a cannonball from 416 feet above sea level with the same velocity. How long is this cannonball in the air?

Scaffolded Practice 3.5.1

Example 1

Joe is trying out long-range archery with his crossbow. He aims at a target 50 meters away. The center of the target is 1 meter high. On his first try, Joe fires the bow straight ahead so that the arrow is perfectly horizontal. The bolt leaves the bow from a height of 1.5 meters and follows a parabolic path to hit the center of the target. Create an equation that models the path of the bolt.

1. Determine the vertex of the parabola.

2. Substitute the vertex into the general equation.

3. Find another point on the parabola.

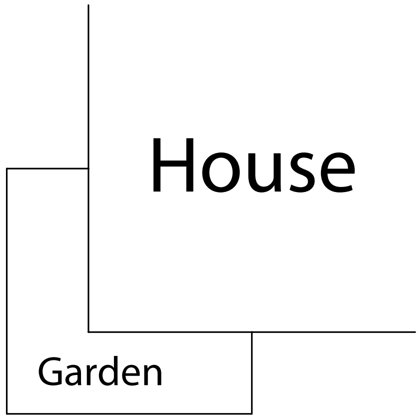
4. Solve for *a*.

5. Find the equation for the path of the bolt.

continued

Example 2

Maurice is building a garden to wrap around the corner of his house. Two sides of the garden will be against the walls of the house, and the other 4 sides will be bordered with bricks. The edges parallel to the corner of the house will be the same length, and the edges perpendicular to the corner of the house will be the same length. Maurice has enough bricks for 20 feet of border. What is the largest garden area that he can enclose with the bricks?



Example 3

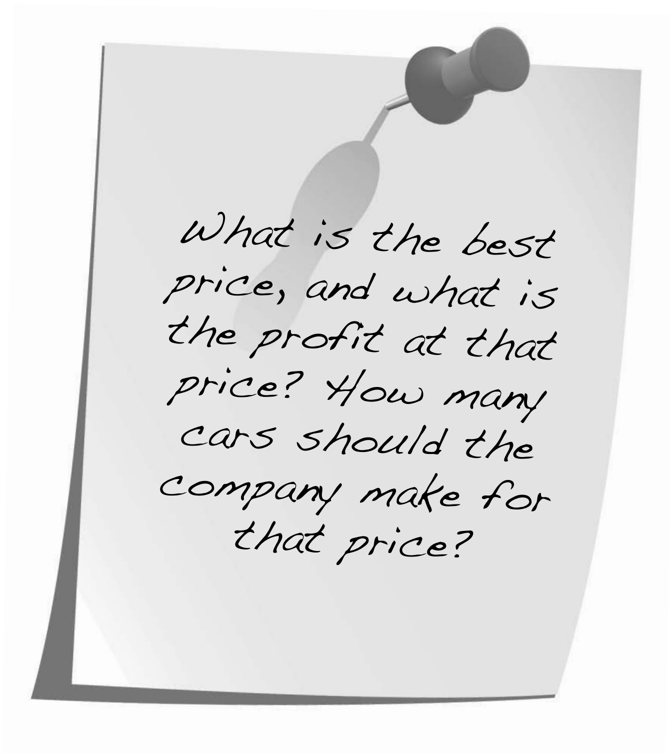
Tianna fires a historical recreation of a demi-cannon across a flat plain in a range test. The cannonball reaches a maximum height of 400 feet and comes to rest 1,600 feet away. Find a quadratic function that models the path of the cannonball. Assume the starting height of the cannonball is 0, and ignore air resistance.

Problem-Based Task 3.5.1: Car Sales

SMP
1 ✓ 2 ✓
3 ✓ 4 ✓
5 6
7 ✓ 8

You have designed a new car model for an automobile manufacturing company. It will cost $7 million to upgrade equipment and pay workers, and it will cost $15,000 to make each car. A market analysis predicts the number of cars sold to relate to the sale price of the car. The function that relates the number of cars sold, $N(x)$, to the sales price, x, is $N(x) = 1{,}000{,}000 - 50x$. The company will use the market analysis to decide how many cars to make and the price at which to sell them.

Create an equation that models the profit from car sales in terms of x. What is the best price, and what is the profit at that price? How many cars should the company make for that price?



Practice 3.5.1: Building Quadratic Functions From Context A

Use your knowledge of quadratic functions to solve problems 1–3.

1. Expand the linear factors of $f(x)$, where $f(x) = (x - 8)(2x + 3)$.

2. Suppose $g(x) = x^2 - 7x + 18$. What is $g(3)$?

3. The product of two consecutive even integers is 3,720. Build a function that can be used to solve for the integers. What are the two integers?

Use the following scenario to complete problems 4 and 5.

> The cables on a suspension bridge form a parabolic shape between the two main towers. The towers extend to 500 feet above the surface of the bridge, and the distance between the tops of the towers is 4,200 feet. At its lowest point, the cable comes within 20 feet of the bridge's road surface. Use the left tower as the *y*-axis and the surface of the bridge as the *x*-axis.

4. What are the coordinates of the vertex and the *y*-intercept?

5. What is the function that describes the curve of the cable on the bridge? Give your answer in vertex form.

continued

Use the following scenario to complete problems 6–7.

> Chen throws a basketball from the 3-point line to the far side of the court. The ball leaves his hand at a height of 6 feet, and it reaches a maximum height of 16 feet at half court. The distance from the 3-point line to half-court is about 17 feet. Use Jonathan's position as the *y*-axis and the floor of the court as the *x*-axis.

6. What are the coordinates of the vertex and *y*-intercept?

7. What is the function that describes the path of the ball? Give your answer in vertex form.

Use the following scenario to complete problems 8–10.

> Kesang is fencing off a section of her yard for her chickens. The coop will be a triangle. One side will be against the wall of her house, but the other 2 sides will be made of fencing and will meet at a 90° angle. Kesang has 16 feet of fencing.

8. If the length of one of the sides of fencing is *x*, what is a function for the area of the coop?

9. What is the largest area Kesang can fence in?

10. How long should Kesang make each side to get the largest coop area?

Practice 3.5.1: Building Quadratic Functions From Context B

Use your knowledge of quadratic functions to solve problems 1–3.

1. Expand the linear factors of $f(x)$, where $f(x) = (5x + 3)(2x + 4)$.

2. Suppose $g(x) = 2x^2 - 3x - 12$. What is $g(10)$?

3. The product of two consecutive integers is 8,372. Build a function that can be used to solve for the integers. What are the two integers?

Use the following scenario to complete problems 4 and 5.

> The cables on a suspension bridge form a parabolic shape between the two main towers. The towers extend to 140 feet above the surface of the bridge, and the distance between the tops of the towers is about 1,600 feet. At its lowest point, the cable comes within 40 feet of the bridge's road surface. Use the left tower as the *y*-axis and the surface of the bridge as the *x*-axis.

4. What are the coordinates of the vertex and the *y*-intercept?

5. What is the function that describes the curve of the cable on the bridge? Give your answer in vertex form.

continued

Use the following scenario to complete problems 6–7.

> Kristine throws a basketball from the foul line to the far side of the court. The ball leaves her hand at a height of 5 feet, and it reaches a maximum height of 20 feet at half court. The distance from the foul line to half-court is about 23 feet. Use Kristine's position as the *y*-axis and the floor of the court as the *x*-axis.

6. What are the coordinates of the vertex and *y*-intercept?

7. What is the function that describes the path of the ball? Give your answer in vertex form.

Use the following scenario to complete problems 8–10.

> Lu is fencing off a section of her yard for her chickens. The coop will be a rectangle. One side will be against the wall of her house, but the other 3 sides will be made of fencing. Lu has 32 feet of fencing.

8. If the length of the side parallel to the wall of the house is *x*, what is a function for the area of the coop?

9. What is the largest area Lu can fence in?

10. How long should Lu make each side to get the largest coop area?

Name: Date:

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Lesson 3.5.2: Comparing Properties of Quadratic Functions Given in Different Forms

Warm-Up 3.5.2

Two kingfishers are perched on a tree branch at the edge of a lake, when they dive into the lake to snatch fish. The given quadratic functions represent the height of each kingfisher relative to the surface of the lake, with $f(x)$ representing the first bird and $g(x)$ representing the second bird. For each set of position functions, determine which bird dives deeper into the lake.

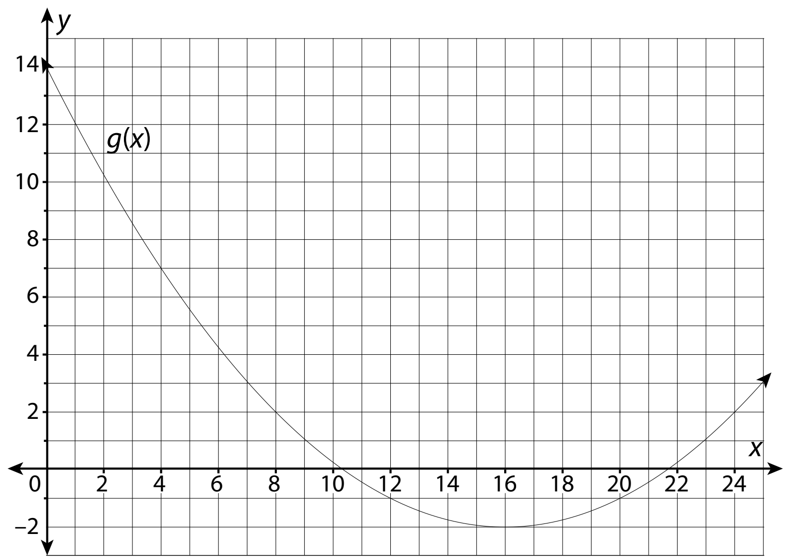
1. $f(x) = \frac{1}{5}(x - 10)^2 - 2$ or $g(x) = \{(0, 15), (10, -1), (20, 15)\}$

2. $f(x) = \frac{1}{10}x^2 - 2x + 9.5$ or the quadratic function that passes through the following points:

x	0	2	4	6	8
$g(x)$	17	7	1	–1	1

3. the quadratic function that passes through the points on the following table or the function described by the following graph:

x	$f(x)$
0	8
5	3
10	0
15	–1
20	0



Scaffolded Practice 3.5.2

Example 1

Which function has the greatest x-intercept, $f(x) = 5x^2 - 35x + 30$ or $g(x) = (x - 3)(x - 4)$?

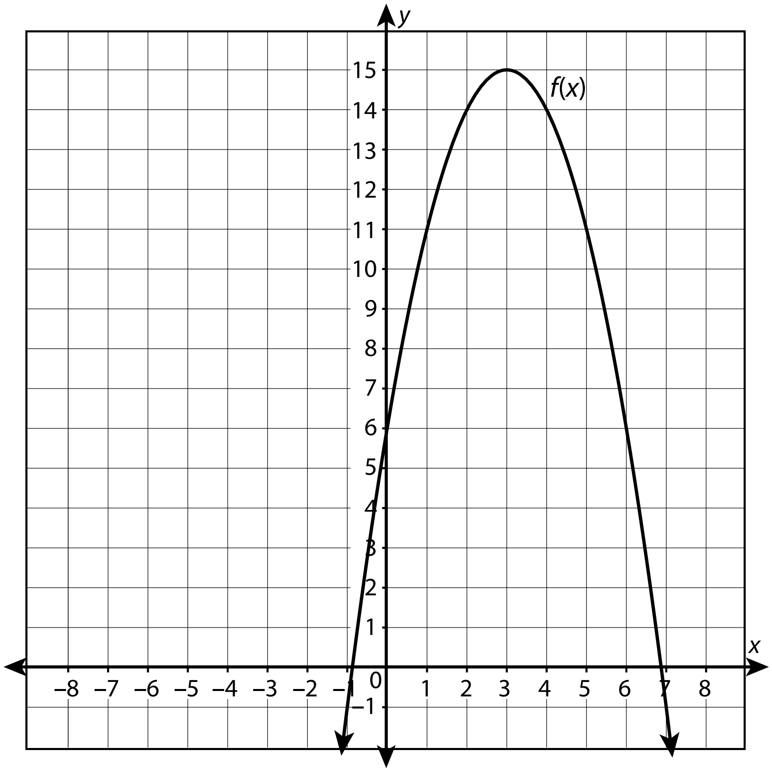
1. Determine the x-intercept of each function.

2. Compare the x-intercepts.

continued

Example 2

Which of the following quadratic functions has the vertex with the larger x-value? The graph of $f(x)$ is shown, and values for $g(x)$ are given in the table.

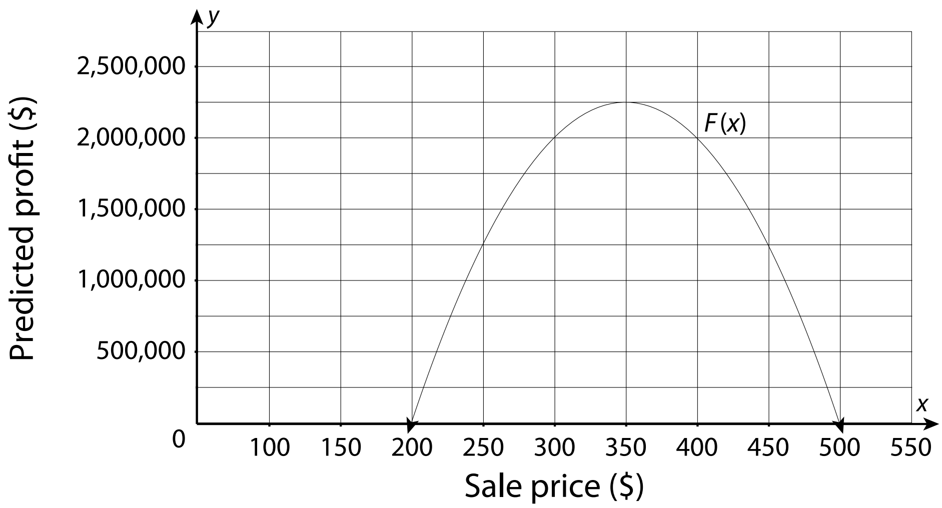


x	$g(x)$
–3	29
–2	24
–1	21
0	20
1	21
2	24
3	29
4	36

continued

Example 3

A small manufacturing company has designed a new product, but the company has to upgrade its equipment to begin production. Before paying for the upgrade, the company wants to be sure it will make enough profit. Two different consultants made profit predictions for the company. Consultant A thinks the profit will follow the function $P(x) = -50x^2 + 42{,}500x - 7{,}500{,}000$, where x is the sale price of the product. Consultant B also thinks the profit is dependent on the sale price of the product, but came up with a different prediction model, $F(x)$. The predicted profit from Consultant B is shown in the following graph. Which consultant predicts a higher maximum profit?



Problem-Based Task 3.5.2: Half-Court Shot

SMP
1 ✓ 2 ✓
3 4
5 6
7 ✓ 8 ✓

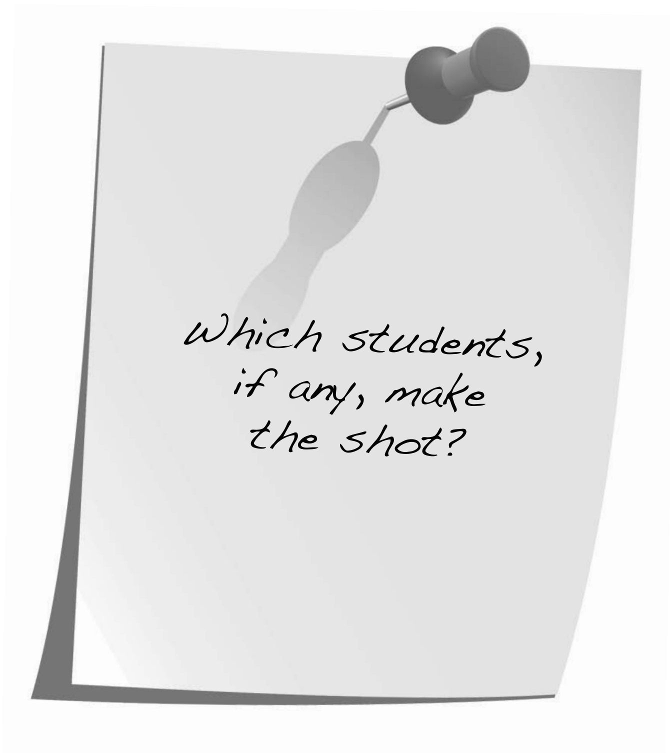
Three students throw basketballs from the center of the court toward one hoop. The height of each student's ball is given as a quadratic function of the horizontal distance it has traveled over the floor. Assume the hoop is located at the point (37, 10). Which students, if any, make the shot?

- The path of Andrea's basketball is modeled by the function $f(x) = -\frac{1}{100}(x-30)^2 + 14$.

- Saul's basketball is estimated to pass through the following points:

x	0	10	20	30
y	3	10.5	13	10.5

- Ichigo's basketball leaves her hand from a height of about 5 feet. When the ball is 12 feet from the basket, it achieves a maximum height of 11.5 feet.



Practice 3.5.2: Comparing Properties of Quadratic Functions Given in Different Forms A

Use the given information to solve problems 1–3.

1. Which function has a lower vertex: a parabola with two x-intercepts and $a > 0$, or a parabola with two x-intercepts and $a < 0$?

2. Which function has a higher y-intercept: a parabola with $a > 0$ and x-intercepts p and $-p$, or a parabola with $a > 0$ and no x-intercepts?

3. Which function's vertex is farther to the right: a parabola with x-intercepts p and $-p$, or a parabola with two x-intercepts that are both less than 0?

Use the following information to complete problems 4–6.

Shelly is testing the braking distance for three different car models. The distance traveled by Model A after she applies the brakes follows the function $A(x) = 2.2x\left(1 + \frac{x}{77}\right)$, where $A(x)$ represents the distance traveled in feet and x represents her speed in miles per hour. The distance traveled (in feet) by Model B is given in the following table. The distance traveled (in feet) by Model C is given by the function $C(x) = 2.2x + \frac{x^2}{26}$.

x	25	35	45	55
***B*(*x*)**	83.4	132.7	191.0	258.5

4. Is there a point after $x = 0$ where two or more models have the same stopping distance?

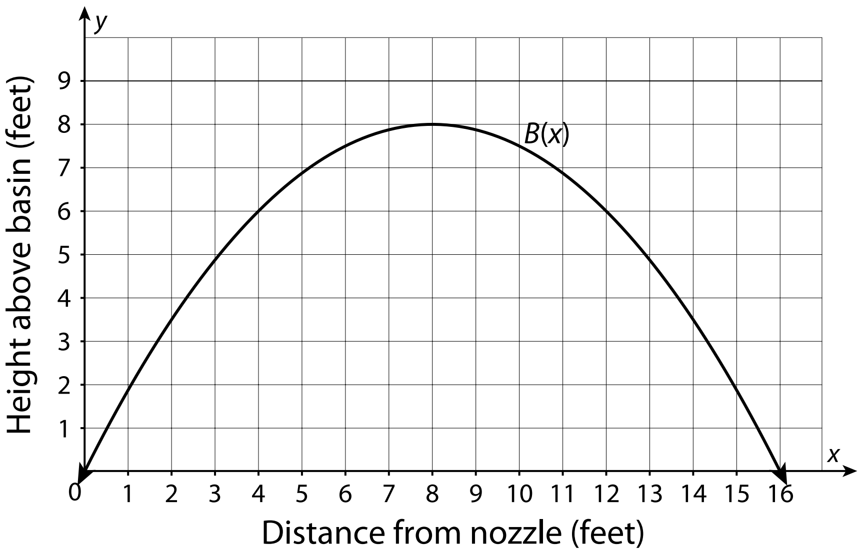
continued

5. Which model has the shortest stopping distance at $x = 25$? Will it always have the shortest stopping distance?

6. Which model has the longest stopping distance at $x = 55$? Will it always have the longest stopping distance?

Use the following information to complete problems 7–10.

A new fountain has been installed in a city park. The fountain has three different types of water jets that stream out of nozzles set in the basin of the fountain. The streams from the water jets all have a quadratic shape. Type A shoots water 12 feet across the basin, and the water stream reaches a maximum height of 12 feet. Type B sprays water along the path shown in the following graph. The path of water shot out of Type C is given in the table.



x (ft)	$C(x)$ (ft)
0	0
2	12
4	16
6	12
8	0

7. Which type reaches the highest point?

continued

8. Which type crosses the longest span?

9. The three nozzle types are arranged in a horizontal line so that the vertexes of the water arcs line up vertically. Suppose the widest arc has its nozzle at (0, 0). Where are the nozzles of each type located?

10. What are the endpoints of each arc?

Practice 3.5.2: Comparing Properties of Quadratic Functions Given in Different Forms B

Use the given information to solve problems 1–3.

1. Which function has a higher vertex: a parabola with two x-intercepts and $a > 0$, or a parabola with no x-intercepts and $a > 0$?

2. Which function has a higher y-intercept: a parabola with $a < 0$ and no x-intercepts, or a parabola with $a > 0$ and no x-intercepts?

3. Which function's vertex is farther to the left: a parabola with x-intercepts p and $-3p$, or a parabola with two x-intercepts that are both greater than 0?

Use the following information to complete problems 4–6.

Sabine is testing the braking distance for three different car models. The distance traveled by Model A after Sabine applies the brakes follows the function $A(x) = 2.2x\left(1+\frac{x}{66}\right)$, where $A(x)$ represents the distance traveled in feet and x represents her speed in miles per hour. The distance traveled by Model B is given in the following table. The distance traveled by Model C is given by the function $C(x) = 2.2x + \frac{x^2}{24}$.

x	25	35	45	55
***B*(*x*)**	73.94	114.12	160.36	212.67

4. Is there a point after $x = 0$ where two or more models have the same stopping distance?

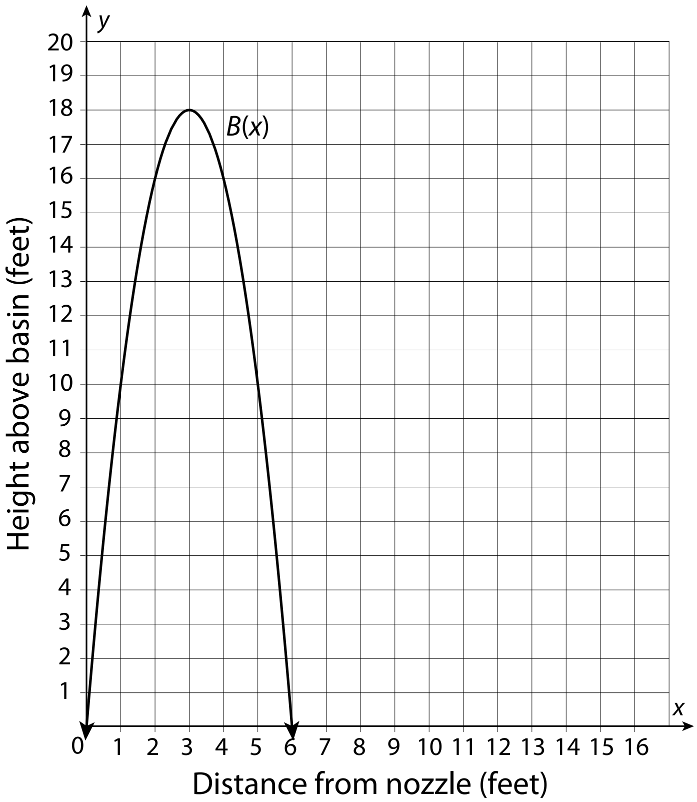
continued

5. Which model has the shortest stopping distance at $x = 25$? Will it always have the shortest stopping distance?

6. Which model has the longest stopping distance at $x = 55$? Will it always have the longest stopping distance?

Use the following information to complete problems 7–10.

A new fountain has been installed in a city park. The fountain has three different types of water jets that stream out of nozzles set in the basin of the fountain. The streams from the water jets all have a quadratic shape. Type A shoots water 10 feet across the basin, and the water stream reaches a maximum height of 10 feet. Type B sprays water along the path shown in the following graph. The path of water shot out of Type C is given in the following table.



x (ft)	$C(x)$ (ft)
0	0
4	3
8	4
12	3
16	0

continued

7. Which type reaches the highest point?

8. Which type crosses the longest span?

9. The three nozzle types are arranged in a horizontal line so that the vertexes of the water arcs line up vertically. Suppose the widest arc has its nozzle at (0, 0). Where are the other two nozzles located?

10. What are the endpoints of each arc?

Name: **Date:**

Notes

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Station 1

At this station, you will find graph paper and a ruler. Work together to graph the following quadratic equation:

$$y = x^2 + 6x + 9$$

1. Write this quadratic equation as a quadratic function.

2. What are the values of *a*, *b*, and *c* in the quadratic function?

   a = _______

   b = _______

   c = _______

To graph the function, you need the vertex, *x*-intercept, and *y*-intercept.

3. If the *x*-value of the vertex is found by $x = \frac{-6}{2(1)} = -3$, then write this *x* calculation using the general terms *a*, *b*, and/or *c*.

4. If the *y*-value of the vertex is found by $y = f\left(\frac{-6}{2(1)}\right) = f(-3) = 0$, then write this *y* calculation using the general terms *a*, *b*, and/or *c*.

5. Based on problems 3 and 4, how can you find the vertex of the graph for $f(x) = ax^2 + bx + c$?

   What is the vertex of the quadratic function $x^2 + 6x + 9 = 0$?

continued

6. How do you find the x-intercept of a function? (*Hint*: $y = f(x)$)

7. How do you find the y-intercept of a function?

8. What are the intercepts for $y = x^2 + 6x + 9$?

9. On your graph paper, graph the function using the vertex, x-intercept, and y-intercept.

10. What shape is the graph? Why do you think the graph has this shape?

Station 2

At this station, you will find a graphing calculator. As a group, follow the steps according to your calculator model to graph $y = x^2 + 4$ and $y = x^2 - 4$.

On a TI-83/84:

Step 1: Press [Y=]. At Y_1, type [X,T,θ,n][x^2][+][4].

Step 2: Press [GRAPH].

On a TI-Nspire:

Step 1: Arrow over to the graphing icon and press [enter]. At *f1*(*x*), enter [*x*], hit the [x^2] key, then type [+][4].

Step 2: Press [enter].

1. What shape is the graph?

2. Does the graph open upward or downward?

3. Which term do you think makes the graph open upward or downward? Explain your reasoning.

On a TI-83/84:

Step 3: Press [2ND], then [GRAPH].

On a TI-Nspire:

Step 3: Press [ctrl], then [T].

4. What information does your calculator show?

5. How can you use this information to find the vertex of the graph?

   What is the vertex of the graph?

continued

On a TI-83/84:	On a TI-Nspire:
Step 4: Press [Y=]. At Y_2, type [X,T,θ,n][x^2] [–][4]. Step 5: Press [GRAPH].	Step 4: Press [ctrl][tab] to go back to the graphing window. Use the touch pad to select ">>" on the bottom left of the screen. At *f2*(*x*), enter [*x*], hit the [x^2] key, then type [–][4]. Step 5: Press [enter].

6. What shape is the graph?

7. Does the graph open upward or downward?

8. Which term do you think makes the graph open upward or downward? Explain your reasoning.

On a TI-83/84:	On a TI-Nspire:
Step 6: Press [2ND], then [GRAPH].	Step 6: Press [ctrl], then [T]. Press [ctrl], then [T] a second time to refresh the screen.

9. What information does your calculator show?

10. How can you use this information to find the vertex of the graph of $y = x^2 - 4$?

    What is the vertex of $y = x^2 - 4$?

11. Why do the graphs for $y = x^2 + 4$ and $y = x^2 - 4$ have different vertices?

Station 3

At this station, you will find a graphing calculator. As a group, follow the steps according to your calculator model to graph $y = x^2$, $y = 3x^2$, and $y = \frac{1}{2}x^2$.

On a TI-83/84:

Step 1: Press [Y=]. At Y_1, type [X,T,θ,n][x^2]. At Y_2, type [3][X,T,θ,n][x^2].

Step 2: Press [GRAPH].

On a TI-Nspire:

Step 1: Arrow over to the graphing icon and press [enter]. At *f1*(*x*), enter [*x*], then hit the [x^2] key. Arrow down. At *f2*(*x*), enter [3][*x*], then hit the [x^2] key.

Step 2: Press [enter].

1. Why do both graphs have the same vertex?

2. Which graph is wider, $y = x^2$ or $y = 3x^2$?

   Why is one graph wider than the other?

On a TI-83/84:

Step 3: Press [2ND], then [GRAPH].

On a TI-Nspire:

Step 3: Press [ctrl], then [T].

3. What information does your calculator show?

4. What is the relationship between Y1 and Y2 in the table?

   How does this relationship relate to $y = x^2$ and $y = 3x^2$?

continued

On a TI-83/84:

Step 4: Press [Y=]. At Y_3, type [0][.][5] [X,T,θ,n][x^2].

Step 5: Press [GRAPH].

On a TI-Nspire:

Step 4: Press [ctrl][tab] to go back to the graphing window. Use the touch pad to select ">>" on the bottom left of the screen. At $f3(x)$, enter [0][.][5][x], then hit the [x^2] key.

Step 5: Press [enter].

5. Why is the graph of $y = 0.5x^2$ wider than $y = x^2$ and $y = 3x^2$?

On a TI-83/84:

Step 6: Press [2ND], then [GRAPH].

On a TI-Nspire:

Step 6: Press [ctrl], then [T]. Press [ctrl], then [T] a second time to refresh the screen.

6. What information does your calculator show?

7. What is the relationship between Y1 and Y3 in the table?

   How does this relationship relate to $y = x^2$ and $y = 0.5x^2$?

8. What is the relationship between Y2 and Y3 in the table?

   How does this relationship relate to $y = 3x^2$ and $y = 0.5x^2$?

Station 4

At this station, you will find graph paper and a ruler. Work together to graph the following quadratic equations:

$f(x) = x^2 - x - 6$ and $f(x) = -x^2 + x - 6$

1. What are the values of a, b, and c in each quadratic function?

$f(x) = x^2 - x - 6$	$f(x) = -x^2 + x - 6$
a = _______	a = _______
b = _______	b = _______
c = _______	c = _______

2. Use the information in problem 1 to find the vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ for each function. Show your work.

3. Find the x-intercepts of $f(x) = x^2 - x - 6$ using factoring. Show your work.

4. On your graph paper, graph $f(x) = x^2 - x - 6$ using its vertex and x-intercepts.

5. Does the parabola open upward or downward? Explain your answer.

continued

6. Fill out the table below to help you graph $f(x) = -x^2 + x - 6$.

x	$y = f(x)$
–4	
0	
4	

Graph $f(x) = -x^2 + x - 6$ on your graph paper.

7. Does the graph open upward or downward? Explain your answer.

8. Will the graph of $f(x) = -x^2 + x - 6$ have x-intercepts? Why or why not?

Station 1

Work as a group to answer the questions. Construct graphs without the aid of a graphing calculator. Show all your work and label the axes of each graph.

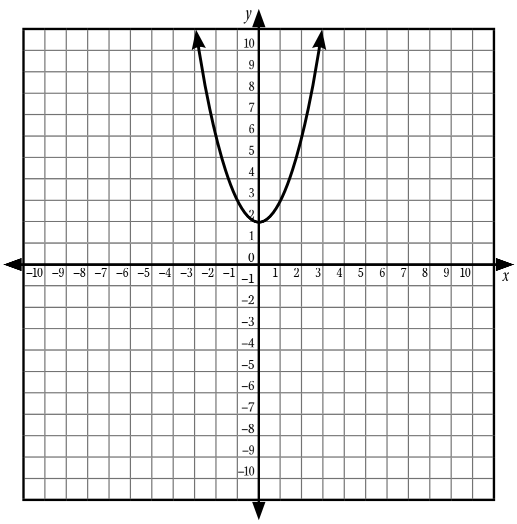
1. Given the equation $y = x^2$, complete the table below with the *y* coordinates for the following values of *x*.

x	*y*
0	
1	
2	
3	
–1	
–2	
–3	

2. Use the coordinates from your table to graph the parabola on graph paper.

3. What are the coordinates for the parabola's *y*-intercept?

4. Look at the parabola below. What is its *y*-intercept?



continued

5. What is the equation for this parabola?

6. How would you write the equation for a similar parabola of y-intercept (0, –5)?

7. Graph the parabola from problem 6.

8. Graph the parabola $y = (x - 2)^2$.

9. What is the equation for the axis of symmetry of this parabola?

10. Without graphing, predict the equation for the axis of symmetry of the parabola $y = (x + 3)^2$.

Station 2

Work with your group to explore the relationship between a quadratic function and its graph.

1. Given the equation $y = 3x^2$, complete the table with the values of y and graph the parabola.

x	y
0	
1	
2	
3	
–1	
–2	
–3	

2. What are the coordinates of this parabola's y-intercept?

3. What is the equation of its axis of symmetry?

4. On the graph from problem 1, draw the parabola $y = x^2$ in a contrasting color. In words, compare the two parabolas.

5. Graph the parabola $y = -2x^2$. Complete the table if you need a reference.

x	y
0	
1	
2	
3	
–1	
–2	
–3	

continued

6. On the same graph, in a contrasting color, graph the parabola $y = -2x^2 + 2$. Label each parabola.

7. What are the coordinates of the *y*-intercept of $y = -2x^2 + 2$?

8. What happens to the graph of a parabola when you add a constant to its equation, as in problem 6?

9. What happens to the graph of a parabola when the x^2 expression is given a coefficient other than 1, as in problems 1 and 5? (*Hint*: Compare the parabola $y = 3x^2$ to $y = x^2$.)

Station 3

Work with your group to answer the following questions.

1. Complete the table for the parabola $y = 2(x - 1)^2 + 3$. Graph the parabola on graph paper.

x	y
0	
1	
2	
3	
4	
–1	
–2	
–3	

2. What is the equation for this parabola's axis of symmetry?

3. What is the vertex?

4. What are the coordinates of this parabola's y-intercept?

5. How would this graph change if the parabola's equation changed to $y = -2(x - 1)^2 + 3$? Graph the new parabola to check your answer.

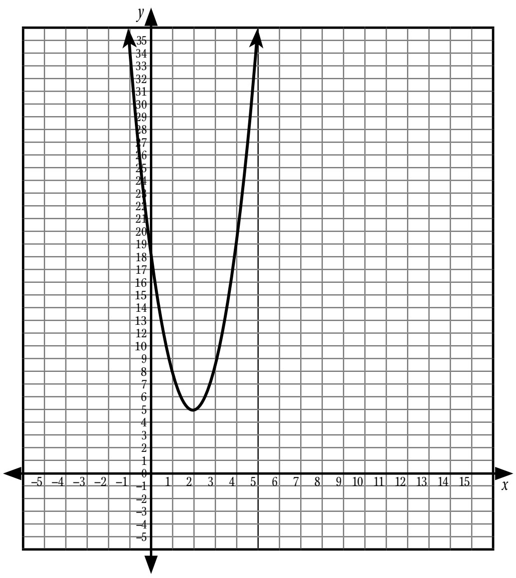
6. What are the coordinates of the y-intercept of the parabola $y = \frac{1}{2}(x - 2)^2 + 1$?

continued

7. What is the vertex?

8. Do you think that the graph of $y = \frac{1}{2}(x - 2)^2$ will be wider or narrower than the graph of $y = (x - 2)^2$? Why? Graph both parabolas, in contrasting colors, to check your answer.

9. Look at the graph below. The equation for this parabola is $y = 3(x - 2)^2 + 5$. What is the vertex? What is the parabola's y-intercept?



10. How would you write the equation for a similar parabola with a y-intercept 5 units higher? Show your work. Write out an explanation in words if necessary.

Station 4

Work with your group to answer the following questions.

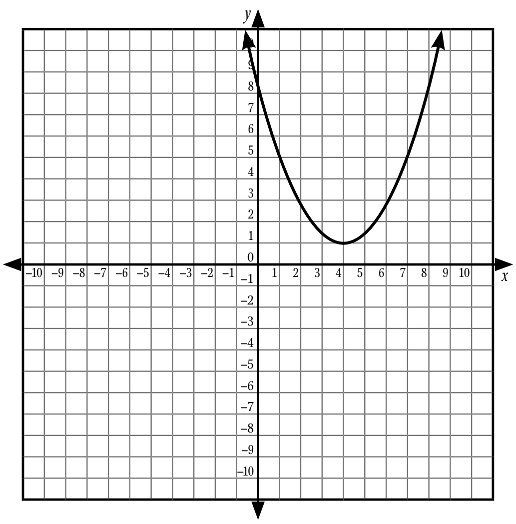
1. Graph the parabola $y = x^2 + 6x + 7$ on graph paper.

2. Give the equation for its axis of symmetry.

3. *Optional*: Complete the square to give the equation for the parabola in vertex form. Show your work.

4. What are the coordinates of the vertex of this parabola?

5. Look at the graph below. What are the coordinates of the vertex of this parabola?



continued

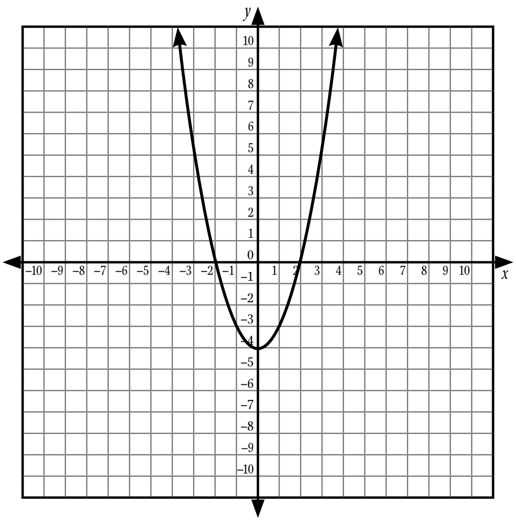
6. The equation for this parabola is $y = \frac{x^2}{2} - 4x + 9$. Use the coordinates found in problem 5 to convert the equation to vertex form. Show your work.

7. What are the coordinates of the *y*-intercept?

8. What is the equation for the axis of symmetry?

9. Does a parabola's axis of symmetry always run through its vertex? Why or why not?

10. Look at the graph below, which shows the parabola $y = x^2 - 4$. The coordinates of the parabola's *x*-intercepts are (2, 0) and (–2, 0). How could you use this information to find the coordinates of the parabola's vertex? Explain, showing your work.



ALGEBRA

Functions	
$f(x)$	Function notation, "*f* of *x*"
$f^{-1}(x)$	Inverse function notation
$f(x) = mx + b$	Linear function
$f(x) = b^x + k$	Exponential function
$(f + g)(x) = f(x) + g(x)$	Addition
$(f - g)(x) = f(x) - g(x)$	Subtraction
$(f \bullet g)(x) = f(x) \bullet g(x)$	Multiplication
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	Division
$\frac{f(b) - f(a)}{b - a}$	Average rate of change
$f(-x) = -f(x)$	Odd function
$f(-x) = f(x)$	Even function
$f(x) = \lfloor x \rfloor$	Floor/greatest integer function
$f(x) = \lceil x \rceil$	Ceiling/least integer function
$f(x) = a\sqrt[3]{(x-h)} + k$	Cube root function
$f(x) = a\sqrt[n]{(x-h)} + k$	Radical function
$f(x) = a\lvert x - h \rvert + k$	Absolute value function
$f(x) = \frac{p(x)}{q(x)};\quad q(x) \neq 0$	Rational function

Symbols	
$\approx$	Approximately equal to
$\neq$	Is not equal to
$\lvert a \rvert$	Absolute value of *a*
$\sqrt{a}$	Square root of *a*
∞	Infinity
[	Inclusive on the lower bound
]	Inclusive on the upper bound
(	Non-inclusive on the lower bound
)	Non-inclusive on the upper bound

Linear Equations	
$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope
$y = mx + b$	Slope-intercept form
$ax + by = c$	General form
$y - y_1 = m(x - x_1)$	Point-slope form

Exponential Equations	
$A = P\left(1 + \frac{r}{n}\right)^{nt}$	Compounded interest formula
Compounded...	*n* (number of times per year)
Yearly/annually	1
Semi-annually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

Quadratic Functions and Equations	
$x = \frac{-b}{2a}$	Axis of symmetry
$x = \frac{p+q}{2}$	Axis of symmetry using the midpoint of the x-intercepts
$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$	Vertex
$f(x) = ax^2 + bx + c$	General form
$f(x) = a(x - h)^2 + k$	Vertex form
$f(x) = a(x - p)(x - q)$	Factored/intercept form
$b^2 - 4ac$	Discriminant
$x^2 + bx + \left(\frac{b}{2}\right)^2$	Perfect square trinomial
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic formula
$(ax)^2 - b^2 = (ax + b)(ax - b)$	Difference of squares
$(x - h)^2 = 4p(y - k)$	Standard form for a parabola that opens up or down
$(y - k)^2 = 4p(x - h)$	Standard form for a parabola that opens right or left
$F(h, k + p)$	Focus for a parabola that opens up or down
$F(h + p, k)$	Focus for a parabola that opens right or left
$y = k - p$	Directrix for a parabola that opens up or down
$x = h - p$	Directrix for a parabola that opens right or left

Exponential Functions	
$1 + r$	Growth factor
$1 - r$	Decay factor
$f(t) = a(1 + r)^t$	Exponential growth function
$f(t) = a(1 - r)^t$	Exponential decay function
$f(x) = ab^x$	Exponential function in general form

General	
(x, y)	Ordered pair
$(x, 0)$	x-intercept
$(0, y)$	y-intercept

Equations of Circles	
$(x - h)^2 + (y - k)^2 = r^2$	Standard form
$x^2 + y^2 = r^2$	Center at (0, 0)
$Ax^2 + By^2 + Cx + Dy + E = 0$	General form

Properties of Radicals
$\sqrt{ab} = \sqrt{a} \bullet \sqrt{b}$
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Imaginary Numbers
$i = \sqrt{-1}$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

Radicals to Rational Exponents
$\sqrt[n]{a} = a^{\frac{1}{n}}$
$\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Multiplication of Complex Conjugates
$(a + bi)(a - bi) = a^2 + b^2$

Properties of Exponents	
Property	General rule
Zero Exponent	$a^0 = 1$
Negative Exponent	$b^{-\frac{m}{n}} = \frac{1}{b^{\frac{m}{n}}}$
Product of Powers	$a^m \bullet a^n = a^{m+n}$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power	$\left(b^m\right)^n = b^{mn}$
Power of a Product	$(bc)^n = b^n c^n$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Formulas

DATA ANALYSIS

Rules and Equations	
$P(E) = \dfrac{\text{\# of outcomes in } E}{\text{\# of outcomes in sample space}}$	Probability of event *E*
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	Addition rule
$P(\overline{A}) = 1 - P(A)$	Complement rule
$P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$	Conditional probability
$P(A \cap B) = P(A) \bullet P(B \mid A)$	Multiplication rule
$P(A \cap B) = P(A) \bullet P(B)$	Multiplication rule if *A* and *B* are independent
${}_nC_r = \dfrac{n!}{(n-r)!r!}$	Combination
${}_nP_r = \dfrac{n!}{(n-r)!}$	Permutation
$n! = n \bullet (n-1) \bullet (n-2) \bullet \cdots \bullet 1$	Factorial

Symbols	
$\varnothing$	Empty/null set
$\cap$	Intersection, "and"
$\cup$	Union, "or"
$\subset$	Subset
$\overline{A}$	Complement of Set A
!	Factorial
${}_nC_r$	Combination
${}_nP_r$	Permutation

GEOMETRY

Symbols

Symbols	
$\overset{\frown}{ABC}$	Major arc length
$\overset{\frown}{AB}$	Minor arc length
$\angle$	Angle
$\odot$	Circle
$\cong$	Congruent
$\overleftrightarrow{PQ}$	Line
$\overline{PQ}$	Line segment
$\overrightarrow{PQ}$	Ray
$\parallel$	Parallel
$\perp$	Perpendicular
$\bullet$	Point
$\triangle$	Triangle
$\square$	Parallelogram
A'	Prime
$^{\circ}$	Degrees
θ	Theta
ϕ	Phi
π	Pi

Area

Area	
$A = lw$	Rectangle
$A=\frac{1}{2}bh$	Triangle
$A=\pi r^2$	Circle
$A=\frac{1}{2}(b_1+b_2)h$	Trapezoid

Trigonometric Ratios

Trigonometric Ratios		
$\sin\theta=\frac{\text{opposite}}{\text{hypotenuse}}$	$\cos\theta=\frac{\text{adjacent}}{\text{hypotenuse}}$	$\tan\theta=\frac{\text{opposite}}{\text{adjacent}}$
$\csc\theta=\frac{\text{hypotenuse}}{\text{opposite}}$	$\sec\theta=\frac{\text{hypotenuse}}{\text{adjacent}}$	$\cot\theta=\frac{\text{adjacent}}{\text{opposite}}$

Trigonometric Identities

Trigonometric Identities
$\sin\theta=\cos(90^\circ-\theta)$
$\cos\theta=\sin(90^\circ-\theta)$
$\tan\theta=\frac{\sin\theta}{\cos\theta}$
$\csc\theta=\frac{1}{\sin\theta}$
$\sec\theta=\frac{1}{\cos\theta}$
$\cot\theta=\frac{1}{\tan\theta}$
$\cot\theta=\frac{\cos\theta}{\sin\theta}$
$\sin^2\theta+\cos^2\theta=1$

Pythagorean Theorem

$a^2 + b^2 = c^2$

Volume

Volume	
$V=lwh$	Rectangular prism
$V=Bh$	Prism
$V=\frac{1}{3}\pi r^2 h$	Cone
$V=\frac{1}{3}Bh$	Pyramid
$V=\pi r^2 h$	Cylinder
$V=\frac{4}{3}\pi r^3$	Sphere

Distance Formula

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Dilation

$$D_k(x,y)=(kx,ky)$$

Pi Defined

$$\pi=\frac{\text{circumference}}{\text{diameter}}=\frac{\text{circumference}}{2\bullet\text{radius}}$$

Circumference of a Circle	
$C = 2\pi r$	Circumference given the radius
$C = \pi d$	Circumference given the diameter

Converting Between Degrees and Radians
$\dfrac{\text{radian measure}}{\pi} = \dfrac{\text{degree measure}}{180}$

Midpoint Formula
$\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Inverse Trigonometric Functions
Arcsin $\theta = \sin^{-1}\theta$
Arccos $\theta = \cos^{-1}\theta$
Arctan $\theta = \tan^{-1}\theta$

Arc Length	
$s = \theta r$	Arc length (θ in radians)

MEASUREMENTS

Length
Metric
1 kilometer (km) = 1000 meters (m)
1 meter (m) = 100 centimeters (cm)
1 centimeter (cm) = 10 millimeters (mm)
Customary
1 mile (mi) = 1760 yards (yd)
1 mile (mi) = 5280 feet (ft)
1 yard (yd) = 3 feet (ft)
1 foot (ft) = 12 inches (in)

Volume and Capacity
Metric
1 liter (L) = 1000 milliliters (mL)
Customary
1 gallon (gal) = 4 quarts (qt)
1 quart (qt) = 2 pints (pt)
1 pint (pt) = 2 cups (c)
1 cup (c) = 8 fluid ounces (fl oz)

Weight and Mass
Metric
1 kilogram (kg) = 1000 grams (g)
1 gram (g) = 1000 milligrams (mg)
1 metric ton (MT) = 1000 kilograms
Customary
1 ton (T) = 2000 pounds (lb)
1 pound (lb) = 16 ounces (oz)

English		Español
	A	
average rate of change the ratio of the difference of output values to the difference of the corresponding input values: $\frac{f(b)-f(a)}{b-a}$; a measure of how a quantity changes over some interval	3.3	**tasa de cambio promedio** proporción de la diferencia de valores de salida a la diferencia de valores correspondientes de entrada: $\frac{f(b)-f(a)}{b-a}$; medida de cuánto cambia una cantidad en cierto intervalo
axis of symmetry of a parabola the line through the vertex of a parabola about which the parabola is symmetric. The equation of the axis of symmetry is $x=\frac{-b}{2a}$.	3.2 3.3	**eje de simetría de una parábola** línea que atraviesa el vértice de una parábola sobre la que la parábola es simétrica. La ecuación del eje de simetría es $x=\frac{-b}{2a}$.
	B	
binomial a polynomial with two terms	3.1	**binomio** polinomio con dos términos
	C	
concave down a graph of a curve that is bent downward, such as a quadratic function with a maximum value	3.3	**cóncavo hacia abajo** gráfico de una curva que se inclina hacia abajo, tal como una función cuadrática con un valor máximo
concave up a graph of a curve that is bent upward, such as a quadratic function with a minimum value	3.3	**cóncavo hacia arriba** gráfico de una curva que se inclina hacia arriba, tal como una función cuadrática con un valor mínimo
concavity with respect to a curve, the property of being arched upward or downward. A quadratic with positive concavity will increase on either side of the vertex, meaning that the vertex is the minimum or lowest point of the curve. A quadratic with negative concavity will decrease on either side of the vertex, meaning that the vertex is the maximum or highest point of the curve.	3.3 3.5	**concavidad** con respecto a una curva, la propiedad de ser arqueado hacia arriba o hacia abajo. Una función cuadrática con concavidad positiva se incrementará en ambos lados del vértice, lo que significa que el vértice es el punto mínimo o más bajo de la curva. Una función cuadrática con concavidad negativa disminuirá a cada lado del vértice, lo que significa que el vértice es el punto máximo o más alto de la curva.

English		Español
	D	
decreasing the interval of a function for which the output values are becoming smaller as the input values are becoming larger	3.3	**decreciente** intervalo de una función por el que los valores de salida se hacen más pequeños a medida que los valores de entrada se hacen más grandes
difference of two squares a pattern for factoring a binomial that consists of two perfect squares that are being subtracted; for example, $x^2 - y^2 = (x + y)(x - y)$	3.1	**diferencia de dos cuadrados** un patrón para factorizar un binomio que consiste en dos cuadrados perfectos que se están restando; por ejemplo
discriminant an expression whose solved value indicates the number and types of solutions for a quadratic. For a quadratic equation in standard form ($ax^2 + bx + c = 0$), the discriminant is $b^2 - 4ac$.	3.1	**discriminante** expresión cuyo valor resuelto indica la cantidad y los tipos de soluciones para una ecuación cuadrática. En una ecuación cuadrática en forma estándar ($ax^2 + bx + c = 0$), el discriminante es $b^2 - 4ac$.
domain the set of all input values (x-values) that satisfy the given function without restriction	3.3	**dominio** conjunto de todos los valores de entrada (valores de x) que satisfacen la función dada sin restricciones
	E	
end behavior the behavior of the graph as x approaches positive infinity and as x approaches negative infinity	3.3	**comportamiento final** el comportamiento de la gráfica al aproximarse x a infinito positivo o a infinito negativo
even function a function that, when evaluated for $-x$, results in a function that is the same as the original function; $f(-x) = f(x)$	3.3	**función par** función que, cuando se la evalúa para $-x$, tiene como resultado una función que es igual a la original; $f(-x) = f(x)$
extrema the minima or maxima of a function	3.3	**extremos** los mínimos o máximos de una función
	F	
factor (noun) one of two or more numbers or expressions that when multiplied produce a given product	3.1	**factor** uno de dos o más números o expresiones que al multiplicarse dan un producto determinado
factor (verb) to write an expression as the product of its factors	3.1	**factorizar** escribir una expresión como el producto de sus factores

English		Español
factored form of a quadratic function the intercept form of a quadratic equation, written as $f(x) = a(x - p)(x - q)$, where p and q are the x-intercepts of the function; also known as the *intercept form of a quadratic function*	3.3	**forma factorizada de una función cuadrática** forma de intercepto de una ecuación cuadrática, se expresa como $f(x) = a(x - p)(x - q)$, en la que p y q son los interceptos de x de la función; también se conoce como la *forma de intercepto de una función cuadrática*
	G	
greatest common factor (GCF) the largest factor that two or more terms share	3.1	**máximo común divisor (GCF)** el factor más grande que comparten dos o más términos
	H	
horizontal compression squeezing of the parabola toward the y-axis	3.4	**compresión horizontal** contracción de la parábola hacia el eje y
horizontal stretch pulling of the parabola and stretching it away from the y-axis	3.4	**estiramiento horizontal** jalar de la parábola y estirarla lejos del eje y
	I	
increasing the interval of a function for which the output values are becoming larger as the input values are becoming larger	3.3	**creciente** intervalo de una función para el que los valores de salida se hacen más grandes a medida que los valores de entrada también se vuelven más grandes
intercept the point at which a line intercepts the x- or y-axis	3.3	**intercepto** punto en el que una línea intercepta el eje x o y
intercept form of a quadratic function the factored form of a quadratic equation, written as $f(x) = a(x - p)(x - q)$, where p and q are the x-intercepts of the function; also known as the *factored form of a quadratic function*	3.2 3.3 3.5	**forma de intercepto de una función cuadrática** forma factorizada de una ecuación cuadrática, expresada como $f(x) = a(x - p)(x - q)$, donde p y q son los interceptos de x de la función; también se conoce como la *forma factorizada de una ecuación cuadrática*
interval the set of all real numbers between two given numbers. The two numbers on the ends are the endpoints. The endpoints might or might not be included in the interval depending on whether the interval is open, closed, or half-open/half-closed.	3.1	**intervalo** conjunto de todos los números reales entre dos números dados. Los dos números en los finales son los extremos. Los extremos podrían o no estar incluidos en el intervalo, según si el intervalo está abierto, cerrado, o medio abierto o medio cerrado.

English		Español
irrational number a number that cannot be written as $\frac{m}{n}$, where m and n are integers and $n \neq 0$; any number that cannot be written as a decimal that ends or repeats	3.1	**números irracionales** un número que no pueden expresarse como $\frac{m}{n}$, en los que m y n son enteros y $n \neq 0$; cualquier número que no puede expresarse como decimal finito o periódico
	K	
key features of a quadratic function the x-intercepts, y-intercept, where the function is increasing and decreasing, where the function is positive and negative, relative minimums and maximums, symmetries, and end behavior of the function used to describe, draw, and compare quadratic functions	3.2 3.3	**características clave de una función cuadrática** interceptos de x, intercepto de y, donde la función aumenta y disminuye, donde la función es positiva y negativa, máximos y mínimos relativos, simetrías y comportamiento final de la función utilizado para describir, dibujar y comparar las funciones cuadráticas
	L	
leading coefficient the coefficient of the term with the highest power. For a quadratic equation in standard form ($y = ax^2 + bx + c$), the leading coefficient is a.	3.1 3.5	**coeficiente líder** coeficiente del término con la mayor potencia. En una ecuación cuadrática en forma estándar ($y = ax^2 + bx + c$), el coeficiente líder es a.
literal equation an equation that involves two or more variables	3.2	**ecuación literal** ecuación que incluye dos o más variables
	M	
maximum the largest y-value of a quadratic equation	3.2 3.3	**máximo** el mayor valor de y de una ecuación cuadrática
minimum the smallest y-value of a quadratic equation	3.2 3.3	**mínimo** el menor valor de y en una ecuación cuadrática
monomial an expression with one term, consisting of a number, a variable, or the product of a number and variable(s)	3.1	**monomio** expresión con un solo término, que consiste en un número, una variable, o el producto de un número y una o más variables
	N	
neither describes a function that, when evaluated for $-x$, does not result in the opposite of the original function (odd) or the original function (even)	3.3	**ni** describe una función que, cuando se evalúa para $-x$, no tiene como resultado lo opuesto de la función original (impar) ni la función original (par)

English		Español
	O	
odd function a function that, when evaluated for $-x$, results in a function that is the opposite of the original function; $f(-x) = -f(x)$	3.3	**función impar** función que, cuando se evalúa para $-x$, tiene como resultado una función que es lo opuesto a la función original; $f(-x) = -f(x)$
	P	
parabola the U-shaped graph of a quadratic equation; the set of all points that are equidistant from a fixed line, called the directrix, and a fixed point not on that line, called the focus. The parabola, directrix, and focus are all in the same plane. The vertex of the parabola is the point on the parabola that is closest to the directrix.	3.2 3.3	**parábola** gráfico de una ecuación cuadrática en forma de U; conjunto de todos los puntos equidistantes de una línea fija denominada directriz y un punto fijo que no está en esa línea, llamado foco. La parábola, la directriz y el foco están todos en el mismo plano. El vértice de la parábola es el punto más cercano a la directriz.
parabolic curve the graph of a quadratic function	3.5	**curva parabólica** la gráfica de una función cuadrática
perfect square an expression that is produced by multiplying a value by itself	3.1	**cuadrado perfecto** una expresión que se produce multiplicando un valor por sí mismo
perfect square trinomial a trinomial of the form $x^2 + bx + \left(\frac{b}{2}\right)^2$ that can be written as the square of a binomial	3.1	**trinomio cuadrado perfecto** trinomio de la forma $x^2 + bx + \left(\frac{b}{2}\right)^2$ que puede expresarse como el cuadrado de un binomio
polynomial a monomial or the sum of monomials	3.1	**polinomio** monomio o suma de monomios
prime an expression that cannot be factored	3.1	**número primo** expresión que no puede ser factorizada
prime factor a factor that is prime	3.1	**factor primario** un factor que es primo
prime number a whole number that can only be evenly divided by itself	3.1	**número primo** un número entero que sólo puede ser dividido por sí mismo por sí mismo

English		Español
	Q	
quadratic equation an equation that can be written in the form $ax^2 + bx + c = 0$, where x is the variable, a, b, and c are constants, and $a \neq 0$	3.1	**ecuación cuadrática** ecuación que se puede expresar en la forma $ax^2 + bx + c = 0$, donde x es la variable, a, b, y c son constantes, y $a \neq 0$
quadratic expression an algebraic expression that can be written in the form $ax^2 + bx + c$, where x is the variable, a, b, and c are constants, and $a \neq 0$	3.1	**expresión cuadrática** expresión algebraica que se puede expresar en la forma $ax^2 + bx + c$, donde x es la variable, a, b, y c son constantes, y $a \neq 0$
quadratic formula a formula that states the solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. A quadratic equation in this form can have no real solutions, one real solution, or two real solutions.	3.1	**fórmula cuadrática** fórmula que establece que las soluciones de una ecuación cuadrática de la forma $ax^2 + bx + c = 0$ están dadas por $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Una ecuación cuadrática en esta forma tener ningún solución real, o tener una solución real, o dos soluciones reales.
quadratic function a function that can be written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. The graph of any quadratic function is a parabola.	3.2 3.3	**función cuadrática** función que puede expresarse en la forma $f(x) = ax^2 + bx + c$, donde $a \neq 0$. El gráfico de cualquier función cuadrática es una parábola.
quadratic inequality an inequality that can be written in the form $ax^2 + bx + c < 0$, $ax^2 + bx + c \leq 0$, $ax^2 + bx + c > 0$, or $ax^2 + bx + c \geq 0$	3.1	**desigualdad cuadrática** desigualdad que puede expresarse en la forma $ax^2 + bx + c < 0$, $ax^2 + bx + c \leq 0$, $ax^2 + bx + c > 0$, o $ax^2 + bx + c \geq 0$
	R	
range the set of all outputs of a function; the set of y-values that are valid for the function	3.3	**rango** conjunto de todas las salidas de una función; conjunto de valores de y que son válidos para la función
rational number any number that can be written as $\frac{m}{n}$, where both m and n are integers and $n \neq 0$; any number that can be written as a decimal that ends or repeats	3.1	**números racionales** números que pueden expresarse como $\frac{m}{n}$, en los que m y n son enteros y $n \neq 0$; cualquier número que puede escribirse como decimal finito o periódico

English		Español
real numbers the set of all rational and irrational numbers	3.1	**números reales** conjunto de todos los números racionales e irracionales
root(s) solution(s) of a quadratic equation	3.1	**raíces** soluciones de una ecuación cuadrática
	S	
slope the measure of the rate of change of one variable with respect to another variable; slope $= m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$; the slope in the equation $y = mx + b$ is m	3.3	**pendiente** medida de la tasa de cambio de una variable con respecto a otra; pendiente $= m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$; la pendiente en la ecuación $y = mx + b$ es m
standard form of a quadratic equation a quadratic equation written as $ax^2 + bx + c = 0$, where x is the variable, a, b, and c are constants, and $a \neq 0$	3.1	**forma estándar de función cuadrática** una ecuación cuadrática expresada como $ax^2 + bx + c = 0$, donde x es la variable, a, b, y c son constantes, y $a \neq 0$
standard form of a quadratic function a quadratic function written as $f(x) = ax^2 + bx + c$, where a is the coefficient of the quadratic term, b is the coefficient of the linear term, and c is the constant term	3.2 3.3 3.5	**forma estándar de función cuadrática** función cuadrática expresada como $f(x) = ax^2 + bx + c$, donde a es el coeficiente del término cuadrático, b es el coeficiente del término lineal, y c es el término constante
	T	
transformation adding or multiplying a constant to a function that changes the function's position and/or shape	3.4	**transformación** suma o multiplicación de una constante con una función que cambia la posición y/o forma de la función
translation transforming a function where the shape and size of the function remain the same but the function moves horizontally and/or vertically; adding a constant to the independent or dependent variable	3.4	**traslación** transformación de una función en la que la forma y el tamaño de la función permanecen iguales pero la función se traslada en sentido horizontal y/o vertical; suma de una constante a la variable independiente o dependiente
trinomial a polynomial with three terms	3.1	**trinomio** polinomio con tres términos

English		Español
	V	
vertex form of a quadratic function a quadratic function written as $f(x) = a(x - h)^2 + k$, where the vertex of the parabola is the point (h, k); the form of a quadratic equation where the vertex can be read directly from the equation	3.2 3.3	**fórmula de vértice de función cuadrática** función cuadrática que se expresa como $f(x) = a(x - h)^2 + k$, donde el vértice de la parábola es el punto (h, k); forma de una ecuación cuadrática en la que el vértice se puede leer directamente de la ecuación
vertex of a parabola the point on a parabola that is closest to the directrix and lies on the axis of symmetry; the point at which the curve changes direction; the maximum or minimum	3.2	**vértice de una parábola** punto en una parábola que está más cercano a la directriz y se ubica sobre el eje de simetría; punto en el que la curva cambia de dirección; el máximo o mínimo
vertical compression squeezing of the parabola toward the *x*-axis	3.4	**compresión vertical** contracción de la parábola hacia el eje *x*
vertical stretch pulling of the parabola and stretching it away from the *x*-axis	3.4	**estiramiento vertical** jalar y estirar la parábola lejos del eje *x*
	X	
***x*-intercept** the point at which the graph crosses the *x*-axis; written as $(x, 0)$	3.2 3.3	**intercepto de *x*** punto en el que el gráfico cruza el eje *x*; se expresa como $(x, 0)$
	Y	
***y*-intercept** the point at which the graph crosses the *y*-axis; written as $(0, y)$	3.2 3.3	**intercepto de *y*** punto en el que el gráfico cruza el eje *y*; se expresa como $(0, y)$
	Z	
Zero Product Property If the product of two factors is 0, then at least one of the factors is 0.	3.1	**Propiedad de producto cero** Si el producto de dos factores es 0, entonces al menos uno de los factores es 0.
zeros the *x*-values of a function for which the function value is 0	3.2	**ceros** valores de *x* de una función para la que el valor de la función es 0