Prerequisite Skills

This lesson requires the use of the following skills:

- multiplying binomials (A–APR.1)
- factoring expressions by the greatest common factor (A–SSE.2)
- factoring monomials and constants (4.OA.4)

Introduction

Knowing how to factor variable expressions is a valuable skill, useful for solving quadratic equations, dividing polynomials, and simplifying radical expressions. Factoring is the opposite of multiplication, and since polynomials are often created by distributing, factoring can often be thought of as reversing distribution to determine the factors that were originally used to create the polynomial. Once an expression has been factored by its greatest common factor, it must be examined to see if it can be factored further.

Key Concepts

Factoring the Difference of Two Squares

- A **perfect square** is an expression that is produced by multiplying a value by itself. When the square root of a perfect square is taken, the result is a whole number or variable.
- Recall that a **binomial** is a polynomial with two terms.
- A binomial consisting of two perfect squares that are being subtracted is known as the **difference of two squares**. The general form is $x^2 - y^2$.
- Such a binomial can be factored as follows: $x^2 - y^2 = (x + y)(x - y)$. Notice that the square root of the first term becomes the first term in both factors, and the square root of the second term becomes the second term in both factors. Also notice that in the first factor the terms are added, and in the other they are subtracted.
- When distributed, $(x + y)(x - y) = x^2 - y^2$.

Factoring Trinomials when *a* = 1

- Recall that a **trinomial** is a polynomial with three terms.
- When a trinomial takes the form $ax^2 + bx + c$, the coefficient a is called the **leading coefficient** because it is the coefficient of the term with the highest power.

- When $a = 1$ in a trinomial of the form $ax^2 + bx + c$, you may be able to factor the trinomial by reversing the distribution. To do this, you must determine which set of factors would create the given trinomial when distributed.
- Use the following steps to factor a trinomial when the value of the leading coefficient (a) is 1:

Factoring a Trinomial with a Leading Coefficient of 1
1. Create two sets of parentheses: (   )(   )
2. Write the variable as the first term in each set of parentheses.
3. Separately list all the possible sets of factors for the constant term (include both positive and negative numbers).
4. Determine which set of factors has a sum that equals the coefficient of the first-degree term.
5. Write those factors, including their signs, as the second terms in both sets of parentheses.
6. Distribute the factors to verify that they result in the original trinomial.

- Note that not all trinomials can be factored.

Factoring Perfect Square Trinomials

- A **perfect square trinomial** is a trinomial that often takes the form $(ax)^2 \pm 2abx + b^2$. Notice that both the first and last terms are perfect squares, and the middle term is twice the product of the square roots of the first and last terms.
- Perfect square trinomials can be factored using the same process for factoring all trinomials. The trinomial can also be factored as follows: $(ax)^2 \pm 2abx + b^2 = (a \pm b)^2$.
- When factoring any polynomial expression, including perfect square trinomials, always check to see if the expression can be factored by the greatest common factor first.

Common Errors/Misconceptions

- neglecting to factor out common factors first
- neglecting to include the common factor after it has been factored
- not taking the square root when using the difference of two squares pattern
- confusing the signs of the terms when factoring trinomials
- incorrectly applying the rules for factoring trinomials