Lesson 3.1.4: Factoring Expressions with $a > 1$

Georgia Standard of Excellence

MGSE9–12.A.SSE.2

Warm-Up 3.1.4 Debrief

Rosalind is painting three rectangular pictures. The first, a portrait, will have an area of $(x^2 + 5x + 4)$ square inches. Next, she will paint a landscape with an area of $(x^2 + 10x + 24)$ square inches. Finally, she plans to create a modern art piece with an area of $(x^2 - 16)$ square inches. Use the formula for the area of a rectangle, $A = lw$, to complete the following problems.

1. If the length of the portrait is $(x + 4)$ inches, what is the portrait's width?

   The portrait will have an area of $(x^2 + 5x + 4)$ square inches.

   Since the area of a rectangle equals its length times its width, and the length is given as $(x + 4)$, factor $x^2 + 5x + 4$ to find the expression that represents the width.

   Write two sets of parentheses and include the variable x as the first term in each factor: $(x \quad)(x \quad)$.

   Determine all possible factors for the constant term, 4:

   $$(2 \bullet 2) \qquad (1 \bullet 4) \qquad (-2 \bullet -2) \qquad (-1 \bullet -4)$$

   Determine which set of factors has a sum of 5 (the coefficient of the first-degree term).

   The set of factors whose sum is 5 is 1 and 4 $(1 + 4 = 5)$.

   Write 1 and 4 as the second terms of the factors:

   $x^2 + 5x + 4 = (x + 1)(x + 4)$

   Double-check these factors by distributing:

   $(x + 1)(x + 4)$

   $= x^2 + 4x + 1x + 4$

   $= x^2 + 5x + 4$

   The factors of the area $x^2 + 5x + 4$ are $(x + 1)$ and $(x + 4)$. Since $(x + 4)$ is the given length, $(x + 1)$ must be the width.

   The width of the portrait is $(x + 1)$ inches.

2. What expressions represent the length and width of the landscape painting, given that the length is longer than the width?

   The landscape painting will have an area of $(x^2 + 10x + 24)$ square inches.

   Since $A = lw$, factor $x^2 + 10x + 24$ to find the expressions that represent the length and width.

   Write two parentheses and include the variable x as the first term in each factor: $(x \quad)(x \quad)$.

   Determine all possible factors for the constant term, 24:

$(1 \bullet 24)$	$(2 \bullet 12)$	$(3 \bullet 8)$	$(4 \bullet 6)$
$(-1 \bullet -24)$	$(-2 \bullet -12)$	$(-3 \bullet -8)$	$(-4 \bullet -6)$

   Determine which set of factors has a sum of 10 (the coefficient of the first-degree term).

   The set of factors whose sum is 10 is 4 and 6 $(4 + 6 = 10)$.

   Write 4 and 6 as the second terms of the factors:

   $x^2 + 10x + 24 = (x + 4)(x + 6)$

   Double-check these factors by distributing:

   $(x + 4)(x + 6)$

   $= x^2 + 6x + 4x + 24$

   $= x^2 + 10x + 24$

   The factors of the area $x^2 + 10x + 24$ are $(x + 4)$ and $(x + 6)$. These expressions represent the length and width of the landscape painting in inches.

3. For the modern art piece, how much larger is the width than the length?

   First, the expressions that represent the length and width of the piece must be found.

   The area of the modern art piece is $(x^2 - 16)$ square inches.

   Since the binomial $x^2 - 16$ meets the conditions of the difference of two squares, it can be factored using the pattern $x^2 - y^2 = (x + y)(x - y)$.

   The square root of x^2 is x; the square root of 16 is 4.

$x^2 - y^2 = (x + y)(x - y)$	Difference of two squares
$(x^2 - 16) = (x + y)(x - y)$	Substitute $x^2 - 16$ for $x^2 - y^2$.
$(x^2 - 16) = [(x) + (4)][(x) - (4)]$	Substitute x for x and 4 for y.
$(x^2 - 16) = (x + 4)(x - 4)$	

The factors of the area $x^2 - 16$ are $(x + 4)$ and $(x - 4)$. These expressions represent the length and width of the modern art piece in inches.

Since it is stated in the problem that the width is the larger of the two dimensions, the width must be $x + 4$, since $x + 4$ will always be larger than $x - 4$.

To determine how much larger the width is than the length, subtract the two values.

$(x + 4) - (x - 4)$	Subtract the factors.
$= x + 4 - x + 4$	Distribute the negative sign.
$= 8$	Combine like terms.

The width is 8 inches larger than the length.

Connection to the Lesson

- Students will apply their understanding of factoring trinomials to factor more advanced trinomials.