

UNIT 3 • MODELING AND ANALYZING QUADRATIC FUNCTIONS

Lesson 3: Interpreting and Analyzing Quadratic Functions

Instruction

Guided Practice 3.3.3

Example 1

Calculate the average rate of change for the function $f(x) = x^2 + 6x + 9$ between $x = 1$ and $x = 3$.

1. Evaluate the function for $x = 3$.

$$f(x) = x^2 + 6x + 9 \quad \text{Original function}$$

$$f(3) = (3)^2 + 6(3) + 9 \quad \text{Substitute 3 for } x.$$

$$f(3) = 36 \quad \text{Simplify.}$$

2. Evaluate the function for $x = 1$.

$$f(x) = x^2 + 6x + 9 \quad \text{Original function}$$

$$f(1) = (1)^2 + 6(1) + 9 \quad \text{Substitute 1 for } x.$$

$$f(1) = 16 \quad \text{Simplify.}$$

3. Use the average rate of change formula to determine the average rate of change between $x = 1$ and $x = 3$.

$$\text{Average rate of change} = \frac{f(b) - f(a)}{b - a} \quad \text{Average rate of change formula}$$

$$\text{Average rate of change} = \frac{f(3) - f(1)}{3 - 1} \quad \text{Substitute 1 for } a \text{ and 3 for } b.$$

$$\text{Average rate of change} = \frac{36 - 16}{2} \quad \text{Substitute the values for } f(3) \text{ and } f(1).$$

$$\text{Average rate of change} = 10 \quad \text{Simplify.}$$

The average rate of change of $f(x) = x^2 + 6x + 9$ between $x = 1$ and $x = 3$ is 10.



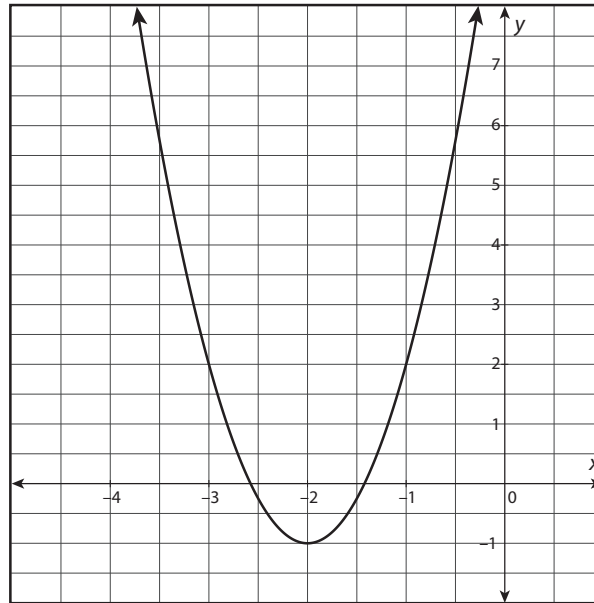
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Example 2

Use the graph of the function to calculate the average rate of change between $x = -3$ and $x = -2$.



1. Use the graph to identify $f(-2)$.

According to the graph, $f(-2) = -1$.

2. Use the graph to identify $f(-3)$.

According to the graph, $f(-3) = 2$.

3. Use the average rate of change formula to calculate the average rate of change between $x = -3$ and $x = -2$.

$$\text{Average rate of change} = \frac{f(b) - f(a)}{b - a} \quad \text{Average rate of change formula}$$

$$\text{Average rate of change} = \frac{f(-2) - f(-3)}{(-2) - (-3)} \quad \text{Substitute } -3 \text{ for } a \text{ and } -2 \text{ for } b.$$

$$\text{Average rate of change} = \frac{-1 - 2}{1} \quad \text{Substitute the values for } f(-3) \text{ and } f(-2).$$

$$\text{Average rate of change} = -3 \quad \text{Simplify.}$$

The average rate of change of the function between $x = -3$ and $x = -2$ is -3 .



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Example 3

For the function $g(x) = (x - 3)^2 - 2$, is the average rate of change greater between $x = -1$ and $x = 0$ or between $x = 1$ and $x = 2$?

1. Calculate the average rate of change between $x = -1$ and $x = 0$.

Evaluate the function at $x = -1$ and $x = 0$.

For $x = -1$:

$$g(x) = (x - 3)^2 - 2$$

$$g(-1) = [(-1) - 3]^2 - 2$$

$$g(-1) = 14$$

For $x = 0$:

$$g(x) = (x - 3)^2 - 2$$

$$g(0) = [(0) - 3]^2 - 2$$

$$g(0) = 7$$

$$\text{Average rate of change} = \frac{g(b) - g(a)}{b - a} \quad \text{Average rate of change formula}$$

$$\text{Average rate of change} = \frac{g(0) - g(-1)}{0 - (-1)} \quad \text{Substitute } -1 \text{ for } a \text{ and } 0 \text{ for } b.$$

$$\text{Average rate of change} = \frac{7 - 14}{1} \quad \text{Substitute the values for } g(-1) \text{ and } g(0).$$

$$\text{Average rate of change} = -7 \quad \text{Simplify.}$$

The average rate of change between $x = -1$ and $x = 0$ is -7 .



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2. Calculate the average rate of change between $x = 1$ and $x = 2$.

Evaluate the function at $x = 1$ and $x = 2$.

For $x = 1$:

$$g(x) = (x - 3)^2 - 2$$

$$g(1) = [(1) - 3]^2 - 2$$

$$g(1) = 2$$

For $x = 2$:

$$g(x) = (x - 3)^2 - 2$$

$$g(2) = [(2) - 3]^2 - 2$$

$$g(2) = -1$$

$$\text{Average rate of change} = \frac{g(b) - g(a)}{b - a} \quad \text{Average rate of change formula}$$

$$\text{Average rate of change} = \frac{g(2) - g(1)}{2 - 1} \quad \text{Substitute 1 for } a \text{ and 2 for } b.$$

$$\text{Average rate of change} = \frac{-1 - 2}{1} \quad \text{Substitute the values for } g(1) \text{ and } g(2).$$

$$\text{Average rate of change} = -3 \quad \text{Simplify.}$$

The average rate of change between $x = 1$ and $x = 2$ is -3 .

3. Compare the averages.

Since $-3 > -7$, the average rate of change of $g(x) = (x - 3)^2 - 2$ is greater between $x = 1$ and $x = 2$ than it is between $x = -1$ and $x = 0$.



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Example 4

Find the average rate of change between $x = -0.75$ and $x = -0.25$ for the following function.

x	$g(x)$
-1	0
-0.75	3.44
-0.5	6.25
-0.25	8.44
0	10
0.25	10.94

1. Identify the output values when $x = -0.75$ and $x = -0.25$.

Refer to the table.

When $x = -0.75$, $g(x) = 3.44$.

When $x = -0.25$, $g(x) = 8.44$.

2. Calculate the average rate of change between $x = -0.75$ and $x = -0.25$ by applying the average rate of change formula.

$$\text{Average rate of change} = \frac{g(b) - g(a)}{b - a} \quad \text{Average rate of change formula}$$

$$\text{Average rate of change} = \frac{g(-0.25) - g(-0.75)}{-0.25 - (-0.75)} \quad \text{Substitute } -0.75 \text{ for } a \text{ and } -0.25 \text{ for } b.$$

$$\text{Average rate of change} = \frac{8.44 - 3.44}{-0.25 - (-0.75)} \quad \text{Substitute the values for } g(-0.75) \text{ and } g(-0.25).$$

$$\text{Average rate of change} = \frac{5}{0.5} \quad \text{Simplify.}$$

$$\text{Average rate of change} = 10$$

The average rate of change of the function between $x = -0.75$ and $x = -0.25$ is 10.

