Guided Practice 4.1.1

Example 1

A population of mice quadruples every 6 months. If a mouse nest started out with 2 mice, how many mice would there be after 2 years? Write an equation and then use it to solve the problem.

1. Read the problem statement carefully and then reread it, this time identifying the known quantities.

   The initial number of mice is 2.

   The base is 4 because the population quadruples each time interval.

   The amount of time is every 6 months for 2 years.

2. Read the statement again, identifying the unknown quantity or variable.

   The unknown quantity is the number of mice after 2 years. Solve for the final amount of mice after 2 years.

3. Create expressions and equations from the known quantities and variable(s).

   The general form of an exponential equation is $y = a \bullet b^x$, where y is the final value, a is the initial value, b is the base, and x is the number of time intervals.

   $a = 2$

   $b = 4$

   Since the problem is given in months, to find x, you need to convert 2 years into 6-month time periods. How many 6-month time periods are there in 2 years?

   To determine this, think about how many 6-month time periods there are in 1 year. There are 2. Multiply that by 2 for each year. Therefore, there are four 6-month time periods in 2 years.

   (continued)

Another way to determine the period is to set up ratios.

$$2\ \cancel{\text{years}} \bullet \frac{12\text{ months}}{1\ \cancel{\text{year}}} = 24\text{ months}$$

$$24\ \cancel{\text{months}} \bullet \frac{1\text{ time period}}{6\ \cancel{\text{months}}} = 4\text{ time periods}$$

Therefore, $x = 4$.

4. Substitute the values into the general form of an exponential equation, $y = a \bullet b^x$.

   $y = a \bullet b^x$ OR $y = ab^{\frac{x}{t}}$

   $y = (2) \bullet (4)^4$ OR $y = (2)(4)^{\frac{24}{6}}$

5. Follow the order of operations to solve the resulting equation.

$y = 2 \bullet 4^4$	Equation from the previous step
$y = 2 \bullet 256$	Raise 4 to the fourth power.
$y = 512$	Multiply 2 and 256.

6. Interpret the solution in terms of the context of the problem.

   There will be 512 mice after 2 years if the population quadruples every 6 months.

Example 2

In sporting tournaments, teams are eliminated after they lose. The number of teams in the tournament then decreases by half with each round. If there are 16 teams left after 3 rounds, how many teams started out in the tournament?

1. Read the problem statement carefully and then reread it, this time identifying the known quantities.

   The final number of teams is 16.

   The reduction is $\frac{1}{2}$ per round.

   The number of rounds is 3.

2. Read the statement again, identifying the unknown quantity or variable.

   The unknown quantity is the number of teams with which the tournament began. Solve for the initial or starting value, a.

3. Create expressions and equations from the known quantities and variable(s).

   The general form of the exponential equation is $y = a \bullet b^x$, where y is the final value, a is the initial value, b is the base (the factor by which it is reduced each round), and x is the number of rounds.

   $y = 16$

   $b = \frac{1}{2}$

   $x = 3$ rounds

4. Substitute the values into the general form of an exponential equation, $y = a \bullet b^x$.

   $y = a \bullet b^x$

   $(16) = a \bullet \left(\frac{1}{2}\right)^{(3)}$

   $16 = a \bullet \left(\frac{1}{2}\right)^{3}$

5. Follow the order of operations to solve the resulting equation.

$16 = a \bullet \left(\frac{1}{2}\right)^3$	Equation from the previous step
$16 = a \bullet \frac{1}{8}$	Raise the base to the power of 3.
$a = 128$	Multiply by the reciprocal of $\frac{1}{8}$.

6. Interpret the solution in terms of the context of the problem.

   The tournament started with 128 teams.

Example 3

The population of a small town is increasing at a rate of 4% per year. If there are currently about 6,000 residents, about how many residents will there be in 5 years at this growth rate?

1. Read the problem statement carefully and then reread it, this time identifying the known quantities.

   The initial number of residents is 6,000.

   The growth rate is 4%.

   The amount of time is 5 years.

2. Read the statement again, identifying the unknown quantity or variable.

   The unknown quantity is the number of residents after 5 years. Solve for the final value after 5 years.

3. Create expressions and equations from the known quantities and variable(s).

   The general form of the exponential growth equation with a percent increase is $y = a(1 + r)^t$, where y is the final value, a is the initial value, r is the rate of growth, and t is the number of years that have elapsed.

   $a = 6000$

   $r = 4\% = 0.04$

   $t = 5$

4. Substitute the values into the general form of an exponential growth equation, $y = a(1 + r)^t$.

   $y = a(1 + r)^t$

   $y = (6000)[1 + (0.04)^{(5)}]$

   $y = 6000(1 + 0.04)^5$

5. Follow the order of operations to solve the resulting equation.

$y = 6000(1 + 0.04)^5$	Equation from the previous step
$y = 6000(1.04)^5$	Add inside the parentheses first.
$y \approx 6000(1.21665)$	Raise the base to the power of 5.
$y \approx 7300$	Multiply.

6. Interpret the solution in terms of the context of the problem.

   If this percentage growth rate continues for 5 years, the population will increase by more than 1,000 residents to about 7,300 people.

Example 4

You want to reduce the size of a picture to place in a small frame. You aren't sure what size to choose on the photocopier, so you decide to reduce the picture by 15% each time you scan it until you get it to the size you want. If the picture was 10 inches long at the start, how long is it after 3 scans?

1. Read the problem statement carefully and then reread it again, this time identifying the known quantities.

   The initial length is 10 inches.

   The reduction is 15% per scan.

   The number of scans is 3.

2. Read the statement again, identifying the unknown quantity or variable.

   The unknown quantity is the length of the picture after 3 scans. Solve for the final value after 3 scans.

3. Create expressions and equations from the known quantities and variable(s).

   The general form of the exponential growth equation with a percent decrease is $y = a(1 - r)^t$, where y is the final value, a is the initial value, r is the rate of decay, and t is the number of intervals.

   $a = 10$

   $r = 15\% = 0.15$

   $t = 3$

4. Substitute the values into the general form of an exponential decay equation, $y = a(1 - r)^t$.

   $y = a(1 - r)^t$

   $y = (10)[1 - (0.15)^{(3)}]$

   $y = 10(1 - 0.15)^3$

5. Follow the order of operations to solve the resulting equation.

$y = 10(1 - 0.15)^3$	Equation from the previous step
$y = 10(0.85)^3$	Subtract inside the parentheses first.
$y = 10(0.614125)$	Raise the base to the power of 3.
$y \approx 6.14$	Multiply.

6. Interpret the solution in terms of the context of the problem.

   After 3 scans, the length of the picture is about 6 inches.