Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- working with exponents (raising a base to a power) (6.EE.1)
- applying the order of operations (5.OA.1)

Introduction

Exponential equations are equations that have the variable in the exponent. Exponential equations are found in science, finance, sports, and many other areas of daily living. Some equations are complicated, but some are not.

Key Concepts

- The general form of an exponential equation is $y = a \bullet b^x$, where a is the initial value, b is the base, and x is the exponent. The final output value will be y.
- Since the equation has an exponent, the value increases or decreases rapidly.
- The base, b, must always be greater than 0 ($b > 0$).
- If the base is greater than 1 ($b > 1$), then the exponential equation represents **exponential growth**.
- If the base is between 0 and 1 exclusive (that is, $0 < b < 1$), then the exponential equation represents **exponential decay**.
- If the time is given in units other than 1 (e.g., 1 month, 1 hour, 1 minute, 1 second), use the equation $y = ab^{\frac{x}{t}}$, where t is the interval over which y changes by a factor of b, and x is the interval under consideration, measured in the same units as t.
- Another form of the exponential equation is $y = a(1 \pm r)^t$, where a is the initial value, $(1 \pm r)$ is the base, t is the variable exponent, and y is the final value.
- Use $y = a(1 + r)^t$ for exponential growth (notice the plus sign). For example, if a population grows by 2% then r is 0.02, but this is less than 1 and by itself does not indicate growth. Adding this to 1 produces the full growth rate, 1.02, or 102%.
- Since r is the percentage increase in a specific time interval, the initial value is multiplied by $(1 + r)$ in each time interval. Therefore, the base $(1 + r)$ indicates exponential growth.

- Use $y = a(1 - r)^t$ for exponential decay (notice the minus sign). For example, if a population decreases by 3%, then 97% is the factor being multiplied over and over again. The population from year to year is always 97% of the population from the year before (a 3% decrease). Think of this as 100% minus the rate, or in decimal form $(1 - r)$.
- Look for words such as *double*, *triple*, *half*, and *quarter*—such words give the number of the base. For example, if an experiment begins with 1 bacterium whose number doubles (splits itself in two) every hour, determining how many bacteria will be present after x hours is solved with the following equation: $y = (1)2^x$, where 1 is the starting value, 2 is the base that indicates doubling, x is the number of hours, and y is the final value.
- Look for the words *initial* or *starting* to find the value to substitute in for a.
- Look for the words *ended with* and *after*—these words will be near the final value given.
- Follow the same procedure as with setting up linear equations and inequalities in one variable:

Creating Exponential Equations from Context

1. Read the problem statement carefully.
2. Reread the scenario and make a list of the known quantities.
3. Read the statement again, identifying the unknown quantity or variable.
4. Create expressions and equations from the known quantities and variable(s).
5. Substitute the values into the general form of an exponential equation.
6. Follow the order of operations to solve the resulting equation.
7. Interpret the solution of the exponential equation in terms of the context of the problem.

Common Errors/Misconceptions

- multiplying a and b together before raising to the power
- misidentifying the base
- creating a linear equation instead of an exponential equation