Instruction

Problem-Based Task 4.1.1: Population Change

Coaching Sample Responses

a. Is Town A experiencing population growth or decay? What is the equation for Town A's population after 5 years?

Town A is experiencing growth. Fill in the information we know to find the equation:

$y = a(1 + r)^t$

- $a = 39{,}000$
- $r = 5\% = 0.05$
- $t = 5$

The equation for Town A is $y = 39{,}000(1 + 0.05)^5$.

b. What is the solution to the equation in part a?

$y = 39{,}000(1 + 0.05)^5$

$y \approx 49{,}775$ people

Round to the nearest whole person.

c. Is Town B experiencing population growth or decay? What is the equation for Town B's population after 5 years?

Town B is experiencing decay. Fill in the information we know to find the equation:

$y = a(1 - r)^t$

- $a = 55{,}000$
- $r = 2\% = 0.02$
- $t = 5$

The equation for Town B is $y = 55{,}000(1 - 0.02)^5$.

d. What is the solution to the equation in part c?

$y = 55{,}000(1 - 0.02)^5$

$y \approx 49{,}716$ people

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e. Are your solutions to parts b and d similar? What can you conclude about the economists' prediction about the populations of the two towns being about the same after 5 years?

The numbers are similar, especially when considering the magnitude. If rounded to the nearest 100 people, Town A would have 49,800 and Town B would have 49,700 people. A difference in 100 people compared to tens of thousands is a small number. Based on these calculations, it would seem that the economists are correct that the populations will be about equal in 5 years.

f. What will the variable, t, equal if the towns experience the same rates of growth or decline for 10 more years after that?

The amount of time passed since the initial population count will be 5 years plus 10, which equals 15 years. Therefore, $t = 15$.

g. What are the equations and solutions for each town's population after 10 more years?

The equations that follow use the town's current populations, with $t = 15$.

Town A	Town B
$y = a(1 + r)^t$	$y = a(1 - r)^t$
$y = 39{,}000(1 + 0.05)^{15}$	$y = 55{,}000(1 - 0.02)^{15}$
$y \approx 81{,}079$ people	$y \approx 40{,}622$ people

Alternatively, students could use the value $t = 10$ and then add the population values they calculated at the 5-year mark for a.

h. Based on your calculations, was the economists' prediction for the town populations after 10 more years correct?

Yes, 81,079 is about twice as much as 40,622.

i. What factors might influence whether or not the economists' predictions come true?

Many factors might contribute to the economists' predictions coming true or not, such as the economy itself. If the economy starts to decline, the population of Town A might not continue to grow as fast. Or, a new industry might relocate to Town B, bringing jobs, and perhaps people will start moving back into that town. Or, Town A might reach its capacity before the population reaches 81,079.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.