Guided Practice 4.1.2

Example 1

If a pendulum swings to 90% of its previous height on each swing and starts out at a height of 60 cm, what is the equation that models this scenario? What is its graph?

1. Read the problem statement and then reread the scenario, identifying the known quantities.

   The initial height is 60 cm.

   The percent remaining after one time period is 90% or 0.90.

2. Substitute the known quantities into the general form of the exponential equation $y = ab^x$.

   In this formula, a is the initial value; b is the amount remaining after each time period, expressed as a decimal; x is the number of intervals (in this case, the number of swings); and y is the final value.

   Let $a = 60$ and $b = 0.90$.

$y = ab^x$	Equation
$y = (60)(0.90)^x$	Substitute 60 for a and 0.90 for b.
$y = 60(0.90)^x$	Simplify.

   The equation $y = 60(0.90)^x$ models the height of the swing.

3. Create a table of values using the resulting equation.

x	y
0	60
1	54
2	48.6
3	43.74
5	35.43
10	20.92
20	7.29
40	0.89

Instruction

4. Set up the coordinate plane.

Determine the labels by reading the problem again. The independent variable is the number of swings. This is the label of the *x*-axis. The *y*-axis label is the height of the pendulum. The height is the dependent variable because it depends on the number of swings.

To determine the scales, examine the table of values. The *x*-axis should have a scale that goes from 0 to 40. Counting to 40 in increments of 1 would cause the axis to be very long or the increments to be very small. Use increments of 5. For the *y*-axis, start with 0 and go to 60 in increments of 5. This will make plotting numbers like 43.74 a little easier than if you chose increments of 10.
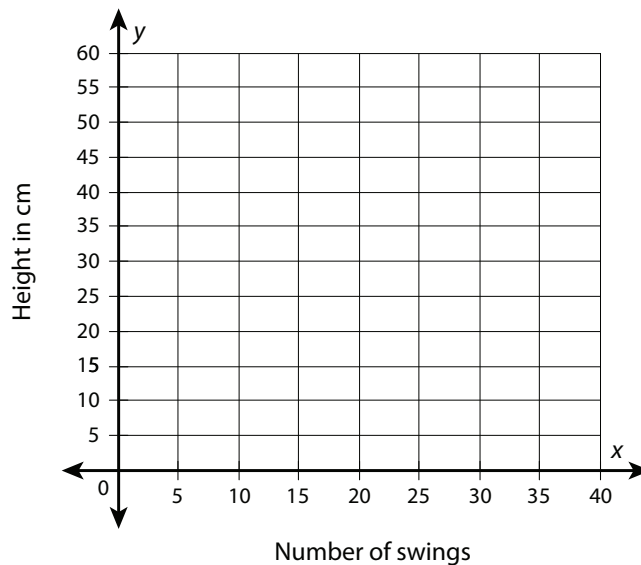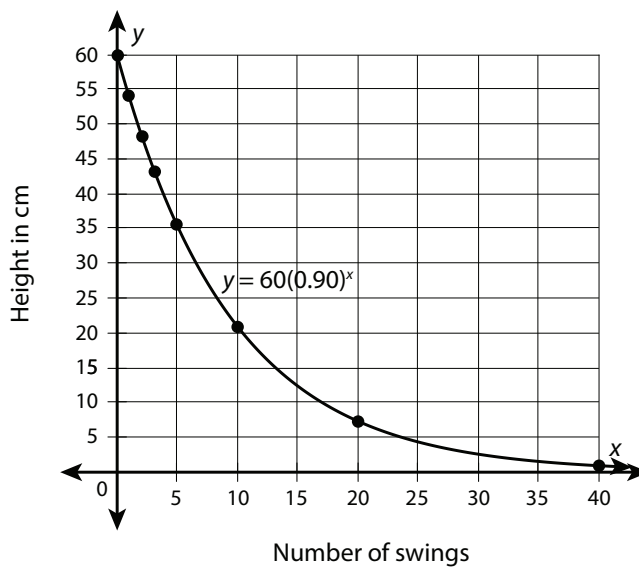
5. Plot the points on the coordinate plane and connect the points with a curve.

    When the points do not lie on a grid line, use estimation to approximate where the point should be plotted. Add an arrow to the right end of the line to show that the curve continues in that direction toward infinity.

$y = 60(0.90)^x$

Height in cm

Number of swings

Example 2

The bacteria *Streptococcus lactis* doubles every 26 minutes in milk. If a container of milk has 4 bacteria in it to start, write an equation that models this scenario and then graph the equation.

1. Read the problem statement and then reread the scenario, identifying the known quantities.

   The initial bacteria count is 4.

   The base that indicates doubling is 2. (If a quantity doubles, there is 200% of the previous quantity, and 200% expressed as a decimal is 2.)

   The time period required for doubling to occur is 26 minutes.

2. Substitute the known quantities into the general form of the exponential equation $y = ab^x$.

   In this formula, a is the initial value, b is the base, x is the number of time intervals (in this case, one time interval is 26 minutes), and y is the final value. Since the time interval is not equal to 1, use the equation $y = ab^{\frac{x}{t}}$, where $a = 4$, $b = 2$, and $t = 26$.

$y = ab^{\frac{x}{t}}$	Equation
$y = (4)(2)^{\frac{x}{(26)}}$	Substitute 4 for a, 2 for b, and 26 for t.
$y = 4(2)^{\frac{x}{26}}$	Simplify.

   The equation $y = 4(2)^{\frac{x}{26}}$ models the number of bacteria.

3. Create a table of values using the resulting equation.

x	y
0	4
26	8
52	16
78	32
104	64

4. Set up the coordinate plane.

   Determine the labels by reading the problem again. The independent variable is the number of time intervals. The time intervals are given in minutes. Therefore, "Minutes" will be the x-axis label. The y-axis label will be the "Number of bacteria." The number of bacteria is the dependent variable because it depends on the number of minutes that have passed.

   The x-axis should have a scale that reflects the fact that each time interval is 26 minutes. The table of values showed 4 time periods. Each time interval is 26 minutes, and so 4 time intervals is $4(26) = 104$ minutes. This means the x-axis scale should go from 0 to 104. Use increments of 26 for easy plotting of the points. For the y-axis, start with 0 and go to 65 in increments of 5. This will make plotting numbers like 32 a little easier than if you chose increments of 10.
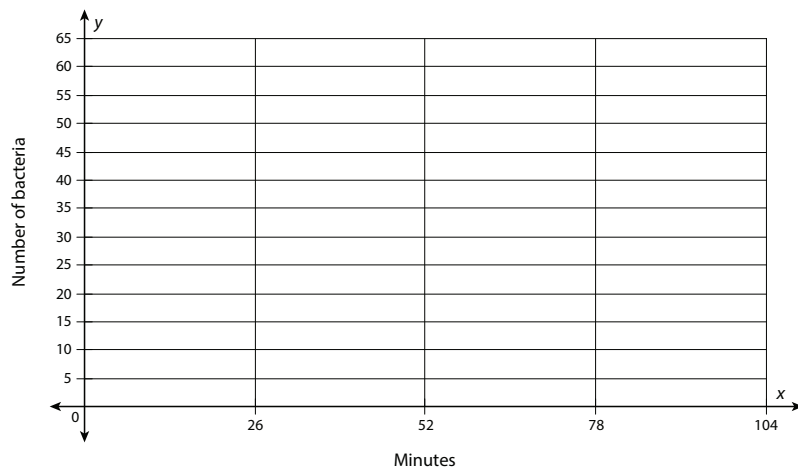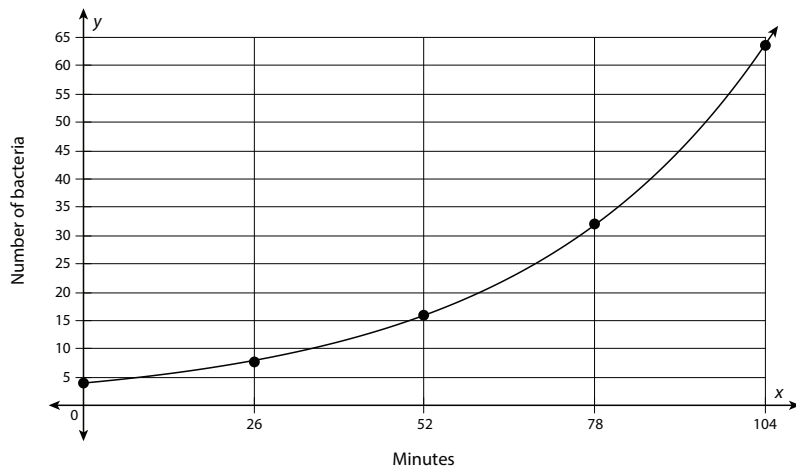
5. Plot the points on the coordinate plane and connect the points with a curve.

   When the points do not lie on a grid line, use estimation to approximate where the point should be plotted. Add an arrow to the right end of the line to show that the curve continues in that direction toward infinity.

Instruction

Example 3

An investment of $500 is compounded monthly at an annual rate of 3%. What is the equation that models the value of the investment? Graph the equation both on a calculator and on graph paper.

1. Read the problem statement and then reread the scenario, identifying the known quantities.

   The initial investment is $500.

   The annual interest rate is 3%.

   Compounded monthly means 12 times per year.

2. Substitute the known quantities into the general form of the compound interest formula, $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$.

   In this formula, P is the initial value, r is the annual interest rate, n is the number of times the investment is compounded in a year, and t is the number of years the investment is left in the account to grow.

   Let $P = 500$, $r = 3\% = 0.03$, and $n = 12$.

   $$A = P\left(1 + \frac{r}{n}\right)^{nt} \qquad \text{Compound interest formula}$$

   $$A = (500)\left[1 + \frac{(0.03)}{(12)}\right]^{(12)t} \qquad \begin{array}{l}\text{Substitute 500 for } P, 0.03 \text{ for } r, \\ \text{and 12 for } n.\end{array}$$

   $$A = 500(1.0025)^{12t} \qquad \text{Simplify.}$$

   Notice that, after simplifying, the resulting equation is of the form

   $y = ab^x$. To graph on the x- and y-axes, rewrite the compound interest

   formula in the form $y = ab^x$, where $A = y$, $P = a$, $\left(1 + \dfrac{r}{n}\right) = b$, and $t = x$.

   $A = 500(1.0025)^{12t}$ becomes $y = 500(1.0025)^{12x}$.

Instruction

3. Graph the equation using a graphing calculator.

On a TI-83/84:

Step 1: Press [Y=].

Step 2: Type in the equation as follows: [500][×][1.0025][^][12][X, T, θ, n]

Step 3: Press [WINDOW] to change the viewing window.

Step 4: At Xmin, enter [0] and arrow down one level to Xmax.

Step 5: At Xmax, enter [10] and arrow down one level to Xscl.

Step 6: At Xscl, enter [1] and arrow down one level to Ymin.

Step 7: At Ymin, enter [500] and arrow down one level to Ymax.

Step 8: At Ymax, enter [700] and arrow down one level to Yscl.

Step 9: At Yscl, enter [15].

Step 10: Press [GRAPH].

On a TI-Nspire:

Step 1: Press the [home] key.

Step 2: Arrow over to the graphing icon and press [enter].

Step 3: At the blinking cursor at the bottom of the screen, enter in the equation [500][×][1.0025][^][12x] and press [enter].

Step 4: To change the viewing window: press [menu], arrow down to number 4: Window/Zoom, and click the center button of the navigation pad.

Step 5: Choose 1: Window settings by pressing the center button.

Step 6: Enter in the appropriate XMin value, [0], and press [tab].

Step 7: Enter in the appropriate XMax value, [10], and press [tab].

Step 8: Leave the XScale set to "Auto." Press [tab] twice to navigate to YMin and enter [500].

Step 9: Press [tab] to navigate to YMax. Enter [700]. Press [tab] twice to leave YScale set to "Auto" and to navigate to "OK."

Step 10: Press [enter].

Step 11: Press [menu] and select 2: View and 5: Show Grid.

Note: To determine the y-axis scale, show the table to get an idea of the values for y. To show the table, press [ctrl] and then [T]. To turn the table off, press [ctrl][tab] to navigate back to the graphing window and then press [ctrl][T] to turn off the table.

4. Transfer your graph from the screen to graph paper.

   Use the same scales that you set for your viewing window.

   The scale on the *x*-axis is from 0 to 10 years in increments of 1 year.

   The scale on the *y*-axis is from $500 to $700 in increments of $15. Show a break in the graph from 0 to 500 with a zigzag line.