Prerequisite Skills

This lesson requires the use of the following skills:

- plotting points in all four quadrants (5.G.1)

- applying the order of operations (5.OA.1)

Introduction

Exponential equations in two variables are similar to linear equations in two variables in that there are an infinite number of solutions. The equation expresses the relationship between the values of the two variables. **Exponential equations** are equations that have a variable in the exponent. This means the y-values of exponential equations are going to increase or decrease very quickly.

Key Concepts

Reviewing Exponential Equations

- The general form of an exponential equation is $y = a \cdot b^x$, where a is the initial value, b is the base, and x is the exponent. The final output value will be y.

- Since the equation has an exponent, the y-value increases or decreases rapidly.

- The base, b, must always be greater than 0 ($b > 0$).

- If the base is greater than 1 ($b > 1$), then the exponential equation represents exponential growth.

- If the base is between 0 and 1 exclusive (that is, $0 < b < 1$), then the exponential equation represents exponential decay.

- If the time is given in units other than 1 (e.g., 1 month, 1 hour, 1 minute, 1 second), use the equation $y = ab^{\frac{x}{t}}$, where t is the interval over which y changes by a factor of b, and x is the interval under consideration, measured in the same units as t. For example, if an initial quantity of 100 doubles every 3 months, the equation would be $y = 100(2)^{\frac{x}{3}}$.

- Another formula for exponential growth or decay is $y = a(1 \pm r)^t$, where a is the initial value, $(1 \pm r)$ is the base, t is the variable exponent, and y is the final value. If $r > 0$, the formula represents growth, and if $0 < r < 1$, the formula represents decay.

Instruction

Introducing the Compound Interest Formula

- The general form of the compounding interest formula is $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$, where A is the initial value, r is the annual interest rate, n is the number of times the investment is compounded in a year, and t is the number of years the investment is left in the account to grow.

- Use this chart for reference:

Compounded…	n (number of times per year)
Yearly/annually	1
Semiannually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

- Remember to change the percentage rate into a decimal by dividing the percentage by 100.

- Apply the order of operations and divide r by n, then add 1. Raise that value to the power of the product nt. Multiply that value by the principal, P.

Graphing Exponential Equations Using a Table of Values

1. Create a table of values by choosing x-values and substituting them into the given equation, then solving for y.

2. Determine the labels by reading the context. The x-axis will most likely be time and the y-axis will be the units of whatever y represents in the problem.

3. Determine the scales. The scale on the y-axis must be large since the values will grow or decline quickly. The scale on the x-axis must be large enough to show the resulting growth or decay.

Instruction

Graphing Equations Using a TI-83/84:

Step 1: Press [Y=] and key in the equation using [^] for the exponent and [X, T, θ, *n*] for *x*.

Step 2: Press [WINDOW] to change the viewing window, if necessary.

Step 3: Enter in appropriate values for Xmin, Xmax, Xscl, Ymin, Ymax, and Yscl, using the arrow keys to navigate.

Step 4: Press [GRAPH].

Graphing Equations Using a TI-Nspire:

Step 1: Press the home key.

Step 2: Arrow over to the graphing icon (the picture of the parabola or the U-shaped curve) and press [enter].

Step 3: At the blinking cursor at the bottom of the screen, enter in the equation using [^] before entering the exponents, and press [enter].

Step 4: To change the viewing window: press [menu], arrow down to number 4: Window/Zoom and click the center button of the navigation pad.

Step 5: Choose 1: Window settings by pressing the center button.

Step 6: Enter in the appropriate XMin, Xmax, YMin, and YMax fields.

Step 7: Leave the XScale and YScale set to auto.

Step 8: Use [tab] to navigate among the fields.

Step 9: Press [tab] to "OK" when done and press [enter].

Common Errors/Misconceptions

- incorrectly applying the order of operations: multiplying *a* and *b* before raising *b* to the exponent in the equation $y = ab^x$

- incorrectly identifying the rate—forgetting to add 1 or subtract from 1

- using the model for exponential growth instead of exponential decay

- forgetting to calculate the number of time periods it takes for a given rate of growth or decay to occur, and simply substituting in the time given