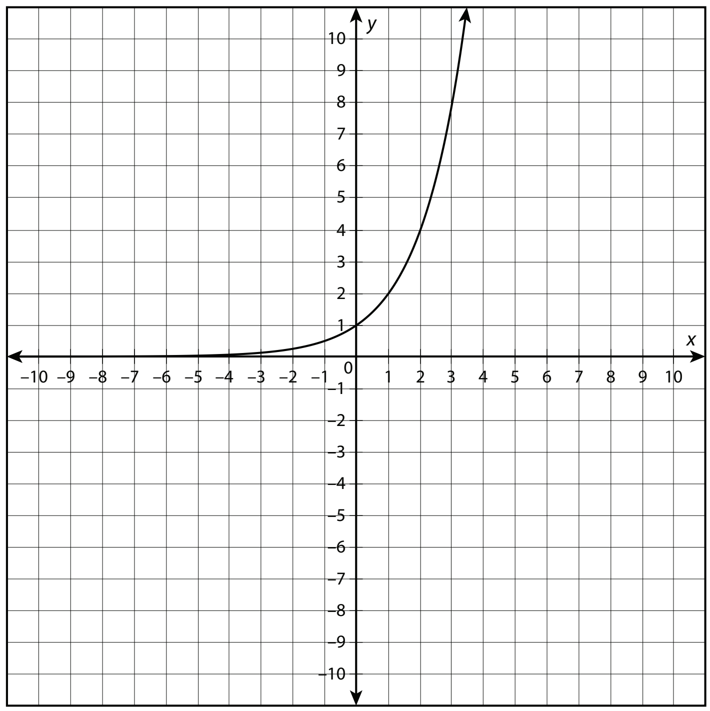
Guided Practice 4.2.1

Example 1

Use the following graph to identify the domain and range of the function $f(x) = 2^x$.



1. Identify the domain.

   The domain is the set of x-values that are valid for the function. This graph goes on infinitely; so, the domain can be any real x-value.

   Domain: {all real numbers}

2. Identify the range.

   The range is the set of y-values that are valid for the function.

   The range will never go below 0. In fact, the range never actually reaches 0. The upper end of the range, however, is limitless.

   Range: $\{y > 0\}$

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Example 2

Identify the domain and range of the function $f(x) = 3 \bullet 6^x - 7$.

1. Identify the domain.

   The domain of an exponential function is all real numbers.

   Therefore, the domain of $f(x) = 3 \bullet 6^x - 7$ is $(-\infty, \infty)$, or {all real numbers}.

2. Identify the range.

   As x increases toward positive infinity, the value of $f(x) = 3 \bullet 6^x - 7$ becomes bigger without limit. So, the upper end of the range is positive infinity.

   As x decreases toward negative infinity, the value of $f(x) = 3 \bullet 6^x - 7$ gets closer and closer to –7:

x	$3 \bullet 6^x - 7$
0	–4
–1	$\frac{1}{2} - 7 = -6.5$
–5	$\frac{1}{2 \bullet 6^4} - 7 \approx -6.9996$
–10	$\frac{1}{2 \bullet 6^9} - 7 \approx -6.99999995$

   Therefore, $f(x) = 3 \bullet 6^x - 7$ gets closer and closer to –7 as x decreases toward negative infinity, although it will never reach –7. This is because the function has a horizontal asymptote at $y = -7$.

   This means that the range of $f(x) = 3 \bullet 6^x - 7$ is $(-7, \infty)$, or $-7 < x < \infty$.

Example 3

A population of bacteria doubles every 4 hours according to the function $f(x) = 32 \bullet 2^{0.25x}$. Find the domain and range of the function.

1. Identify the domain.

   An exponential function has a domain of all real numbers, but we must consider the context of the problem. Because time cannot be negative, the domain is all real numbers greater than or equal to 0.

   So, the domain of $f(x) = 32 \bullet 2^{0.25x}$ is $[0, \infty)$, or $x \geq 0$.

2. Identify the range.

   The range is the set of $f(x)$ values that are valid for the function. Because the function gets bigger as x increases, the smallest value will occur at $x = 0$. The upper end of the range is limitless.

   To find the lower end of the range, evaluate $f(x) = 32 \bullet 2^{0.25x}$ for $x = 0$:

$f(x) = 32 \bullet 2^{0.25x}$	Original function
$f(0) = 32 \bullet 2^{0.25(0)}$	Substitute 0 for x.
$f(0) = 32 \bullet 2^{0}$	Simplify.
$f(0) = 32 \bullet 1$	Evaluate the exponent.
$f(0) = 32$	Simplify.

   Therefore, the range of $f(x) = 32 \bullet 2^{0.25x}$ is $(32, \infty)$, or $f(x) \geq 32$.

Example 4

A certificate of deposit, or CD, earns interest faster than a regular savings account. However, once you open a CD, you can't add or withdraw any money from it for a set number of years. Rocky has $3,000 that he plans to put in a CD. The CD pays 8% interest, compounded yearly, and lasts 5 years. At the end of the 5 years, Rocky plans to put the money from the CD into a regular savings account that already holds $500 but earns no interest. The total money Rocky has in both the CD and his savings account is represented by the function $f(x) = 3000 \bullet 1.08^x + 500$. Identify the domain and range for this situation. Will Rocky eventually have more than $4,000 in total? If so, starting when?

1. Identify the domain.

   The domain will begin at $x = 0$, because time cannot be negative. It will end at $x = 5$ because the CD ends after 5 years. Additionally, the domain includes only whole numbers, because the CD pays interest yearly.

   So, the domain for this situation is {0, 1, 2, 3, 4, 5}.

2. Identify the range.

   The range of the function will be the outputs for the domain values. To find the range, evaluate $f(x) = 3000 \bullet 1.08^x + 500$ for every term of the domain. The results are compiled in the following table:

x	$f(x)$
0	$3000 \bullet 1.08^{(0)} + 500 = \$3,500$
1	$3000 \bullet 1.08^{(1)} + 500 = \$3,740$
2	$3000 \bullet 1.08^{(2)} + 500 = \$3,999.20$
3	$3000 \bullet 1.08^{(3)} + 500 \approx \$4,279.14$
4	$3000 \bullet 1.08^{(4)} + 500 \approx \$4,581.47$
5	$3000 \bullet 1.08^{(5)} + 500 \approx \$4,907.98$

   So, the range for this situation is {3500, 3740, 3999.20, 4279.14, 4581.47, 4907.98}.

3. Determine whether Rocky will ever have more than \$4,000. If he does, identify the point at which this happens.

   In the previous step, we found the range is {3500, 3740, 3999.20, 4279.14, 4581.47, 4907.98}. The fourth entry in the range, \$4,279.14, is greater than \$4,000, so we know Rocky will eventually have more than \$4,000 in savings. This value of $f(x)$ corresponds to $x = 3$.

   Rocky will have more than \$4,000 after 3 years, at which point he will have \$4,279.14 in total.