Instruction

Prerequisite Skills

This lesson requires the use of the following skills:

- substituting values for variables (7.EE.4)
- applying the order of operations to simplify expressions (7.EE.1)
- evaluating exponents (8.EE.1)

Introduction

Because we have already studied linear and quadratic functions, we can now begin working with exponential functions. In this lesson, we will explore the domain and range of exponential functions, as well as practice evaluating exponential functions. In particular, we will practice identifying the domain and range of functions that represent a real-world situation.

For example, say you deposit $2,000 into a savings account that pays 3% interest every year, and you withdraw all of your money after 5 full years. The amount of money in your account grows according to the function $f(x) = 2000 \cdot 1.03^x$, where x is measured in years. What are the possible values of x and $f(x)$ for this situation?

You wouldn't use negative values for x, since the situation starts when you deposit the money, at $x = 0$. If you took all of your money out of the account after 5 years, you also wouldn't use values of x greater than 5, because the situation no longer applies after this time. Also, you would only look at whole numbers for x, since the value of the account increases only once per year. You would also only consider $f(x)$ values for whole values of x, and you wouldn't look at $f(x)$ values for $x < 0$ or $x > 5$. In other words, the domain of the function is {0, 1, 2, 3, 4, 5}, and the range of the function is $\{f(0), f(1), f(2), f(3), f(4), f(5)\}$.

Because of the limits of the real-world situation, this domain and range is significantly different from the mathematical domain and range of $f(x) = 2000 \cdot 1.03^x$. Without the real-world context, the domain of $f(x) = 2000 \cdot 1.03^x$ is all real numbers, and the range is $f(x) > 0$.

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Key Concepts

- Recall that the **domain** of a function is the set of all inputs, or x-values, that are valid for the function.
- Recall that the **range** of a function is the set of all outputs, or $f(x)$ values, that are valid for the function.
- Recall that to evaluate a function $f(x)$, you must substitute values of x for each variable x in the function.
- An **exponential function** is a function that has a variable in the exponent. The general form is $f(x) = ab^x + k$, where a is the initial value, $b \neq 1$ is the base, and k is a constant value.
- A **horizontal asymptote** is a horizontal line that a function gets closer and closer to as x increases or decreases without bound. Every exponential function with a domain of all real numbers has a horizontal asymptote. The asymptote will be the line $y = k$, where k is the constant in the exponential function.
- The domain of an exponential function with no restrictions is all real numbers.
- The range of an exponential function with no restrictions is a subset of the real numbers. One end of the range will be k; the other end of the range will be infinite.
- For example, find the range of the function $f(x) = -1 \cdot 3^x + 1$.
- As x increases, the function becomes more and more negative. You can determine this by evaluating the function for increasingly large values of x:

x	$f(x)$
0	0
1	–2
5	–242
10	–59,048

- As x decreases, the function gets closer and closer to 1, although it will never reach 1 exactly:

x	$f(x)$
0	0
–1	–0.66667
–5	0.99588
–10	0.99998

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- The range of the function is $(-\infty, 1)$, or $f(x) < 1$.
- You can also identify the domain and range of an exponential function by looking at its graph.
- The domain will be all valid x-values on the graph. If the function continues off the edge of the graph (if the graph has arrows on the ends), the domain is all real numbers.
- The range will be all valid y-values on the graph. If the function continues off the edge of the graph, the range will be all $f(x)$ values spanned between the asymptote value and infinity. Note that the infinity could be negative, depending on the shape of the graph.

Common Errors/Misconceptions

- thinking that $f(x)$ means $f \cdot x$
- not understanding how to use the domain to find the range
- confusing the domain and range of a function