Instruction

Problem-Based Task 4.2.1: Pine Trees upon the Highland

Coaching Sample Responses

a. What does $f(x)$ represent?

$f(x)$ represents the maximum height of a tree at age x.

b. What does $g(x)$ represent?

$g(x)$ represents the minimum height of a tree at age x.

c. What is the domain of $f(x)$ and $g(x)$?

Because $f(x) = -140 \cdot 1.006^{-x} + 140$ and $g(x) = -90 \cdot 1.009^{-x} + 90$ represent the maximum and minimum heights of a tree, the domain will begin at 0, when the tree starts growing, and end at the death of the tree. We don't know when this is, so we will let the end of the domain be infinity.

The domain of $f(x)$ and $g(x)$ is $(0, \infty)$, or $x \geq 0$.

d. What is the range of $f(x)$?

The lower end of the range of $f(x) = -140 \cdot 1.006^{-x} + 140$ is 0 feet, which is the height of a tree at time $x = 0$. As x increases, notice that the value of the exponential term, $-140 \cdot 1.006^{-x}$, grows closer to 0:

x	$f(x) = -140 \cdot 1.006^{-x} + 140$
0	$-140 \cdot 1.006^{(0)} + 140 = -140 + 140 = 0$
100	$-140 \cdot 1.006^{(-100)} + 140 \approx -76.97 + 140 \approx 63.08$
200	$-140 \cdot 1.006^{(-200)} + 140 \approx -42.32 + 140 \approx 97.68$
300	$-140 \cdot 1.006^{(-300)} + 140 \approx -23.27 + 140 \approx 116.73$
400	$-140 \cdot 1.006^{(-400)} + 140 \approx -12.79 + 140 \approx 127.21$

It appears that the value of the function will get increasingly closer to its constant term, 140, as x gets increasingly bigger.

Therefore, the range of $f(x)$ is $(0, 140)$, or $0 \leq f(x) < 140$.

e. What is the range of $g(x)$?

The lower end of the range of $g(x) = -90 \cdot 1.009^{-x} + 90$ is 0 feet, which is the height of a tree at time $x = 0$. As x increases, notice that the exponential term, $-90 \cdot 1.009^{-x}$, becomes less negative:

x	$f(x) = -90 \cdot 1.009^{-x} + 90$
0	$-90 \cdot 1.009^{(0)} + 90 \approx -90 + 90 = 0$
100	$-90 \cdot 1.009^{(-100)} + 90 \approx -36.74 + 90 \approx 53.26$
200	$-90 \cdot 1.009^{(-200)} + 90 \approx -15.00 + 90 \approx 75.00$
300	$-90 \cdot 1.009^{(-300)} + 90 \approx -6.12 + 90 \approx 83.88$
400	$-90 \cdot 1.009^{(-400)} + 90 \approx -2.50 + 90 \approx 87.50$

It appears that the value of the exponential term is getting increasingly closer to 0. Therefore, the function will get increasingly closer to its constant term, 90, as x gets increasingly bigger.

The range of $g(x)$ is (0, 90), or $0 \leq x < 90$.

f. What is the shortest a 400-year-old tree could be?

To find the shortest a 400-year-old tree could be, evaluate the minimum height function, $g(x)$, for $x = 400$.

$g(x) = -90 \cdot 1.009^{-x} + 90$

$g(400) = -90 \cdot 1.009^{-400} + 90$

$g(400) \approx -2.50 + 90$

$g(400) \approx 87.50$

The shortest a 400-year old tree could be based on the minimum height function is about 87.50 feet.

g. Which trees could be more than 400 years old?

Select the trees from the table that are more than 87.50 feet tall.

Tree 1	96 feet
Tree 2	72 feet
Tree 3	120 feet
Tree 4	139 feet
Tree 5	80 feet

Tree 1, Tree 3, and Tree 4 could be more than 400 years old.

h. Which tree do you think is oldest?

The oldest trees could vary in height between 90 and 140 feet. However, the tallest one in the grove could be the oldest. The tallest tree is Tree 4, at 139 feet.

i. What is the tallest a tree could be after 400, 500, 600, 700, and 800 years?

To find the tallest a tree could be after each number of years, evaluate the maximum height function $f(x) = -140 \cdot 1.006^{-x} + 140$ for each value of x. The following table shows the results of these calculations:

x	$f(x)$
400	127.21
500	132.97
600	136.13
700	137.87
800	138.83

j. How old is the oldest tree?

We decided that the tallest tree, Tree 4, is probably the oldest. Based on the maximum height function, the oldest tree is probably more than 800 years old.

Recommended Closure Activity

Select one or more of the essential questions for a class discussion or as a journal entry prompt.