Guided Practice 4.3.1

Example 1

Find the common ratio, write the explicit formula, and find the seventh term for the following geometric sequence.

3, 1.5, 0.75, 0.375, ...

1. Find the common ratio by dividing two successive terms.

   $1.5 \div 3 = 0.5$

2. Confirm that the ratio is the same between each remaining pair of consecutive terms.

   $0.75 \div 1.5 = 0.5$ and $0.375 \div 0.75 = 0.5$

   $r = 0.5$

3. Identify the first term, a_1.

   $a_1 = 3$

4. Write the explicit formula.

$a_n = a_1 \bullet r^{n-1}$	Explicit formula for a geometric sequence
$a_n = (3)(0.5)^{n-1}$	Substitute 3 for a_1 and 0.5 for r.

5. To find the seventh term, substitute 7 for n.

$a_n = (3)(0.5)^{n-1}$	Explicit formula from the previous step
$a_{(7)} = (3)(0.5)^{(7)-1}$	Substitute 7 for n.
$a_7 = (3)(0.5)^6$	Simplify.
$a_7 = (3)(0.015625)$	Raise 0.5 to the 6th power.
$a_7 = 0.046875$	Multiply.

   The seventh term in the sequence is 0.046875.

SWB p. 49

Example 2

The fifth term of a geometric sequence is 1,792. The common ratio is 4. Write an explicit formula for the sequence, and then write the corresponding exponential function.

1. The fifth term is 1,792; therefore $n = 5$ and $a_5 = 1792$.

2. The common ratio is 4; therefore, $r = 4$.

3. Substitute the known values into the explicit formula for a geometric sequence and solve for a_1.

$a_n = a_1 \bullet r^{n-1}$	Explicit formula for a geometric sequence
$(1792) = a_1(4)^{(5)-1}$	Substitute 1,792 for a_n, 4 for r, and 5 for n.
$1792 = a_1(256)$	Simplify.
$a_1 = 7$	

4. Write the explicit formula.

$a_n = a_1 \bullet r^{n-1}$	Explicit formula for a geometric sequence
$a_n = (7)(4)^{n-1}$	Substitute 7 for a_1 and 4 for r.

5. Write the explicit formula in function notation.

$f(x) = 7(4)^{x-1}$

Note that the domain of a geometric sequence is consecutive positive integers.

Example 3

A geometric sequence is defined recursively by $a_n = (a_{n-1})\left(-\frac{1}{3}\right)$, with $a_1 = 729$. Find the first 5 terms of the sequence, write an explicit formula to represent the sequence, then find the eighth term.

1. Use the recursive formula, beginning with a_1, to calculate the next 4 terms.

   We are given that the first term, a_1, is 729. Substitute 2, 3, 4, and 5, respectively, for n in the recursive formula $a_n = (a_{n-1})\left(-\frac{1}{3}\right)$ to find the next 4 terms.

   $$a_1 = 729$$
   $$a_2 = (729)\left(-\frac{1}{3}\right) = -243$$
   $$a_3 = (-243)\left(-\frac{1}{3}\right) = 81$$
   $$a_4 = (81)\left(-\frac{1}{3}\right) = -27$$
   $$a_5 = (-27)\left(-\frac{1}{3}\right) = 9$$

   The first five terms of the sequence are 729, –243, 81, –27, and 9.

2. Write the explicit formula for this sequence.

   The first term is $a_1 = 729$ and the common ratio is $r = -\frac{1}{3}$, so the explicit formula is $a_n = (729)\left(-\frac{1}{3}\right)^{n-1}$.

3. Use the formula to find the requested term.

   We are asked to find the eighth term in this sequence. Substitute 8 for *n* and evaluate.

$a_n = (729)\left(-\frac{1}{3}\right)^{n-1}$	Explicit formula from the previous step
$a_{(8)} = (729)\left(-\frac{1}{3}\right)^{(8)-1}$	Substitute 8 for *n*.
$a_8 = (729)\left(-\frac{1}{3}\right)^{7}$	Simplify.
$a_8 = (729)\left(-\frac{1}{2187}\right)$	Raise $-\frac{1}{3}$ to the 7th power.
$a_8 = -\frac{1}{3}$	Multiply.

   The eighth term of the sequence is $-\frac{1}{3}$.