Guided Practice 4.7.1

Example 1

Adrina puts \$5,000 into a savings account. The account accumulates 0.3% interest every month. How much money is in her account after 1, 2, and 3 months? Find an explicit function to represent the balance in her account at any month.

1. Use the description of the account balance to find the balance after each month.

   Adrina's account has \$5,000. After 1 month, she will have 0.3% more in the account, so her account balance will increase by 0.3%.

   $\$5,000 + 0.003 \bullet \$5000 = \$5015$

   The new starting balance of Adrina's account is \$5,015. After month 2, the interest is compounded again, so she will add 0.3% of the account balance to the value of the account.

   $\$5015 + 0.003 \bullet \$5015 \approx \$5030.05$

   The new starting balance of Adrina's account is \$5,030.05. After month 3, the interest is compounded again, so she will add 0.3% of the account balance to the value of the account.

   $\$5030.05 + 0.003 \bullet \$5030.05 \approx \$5045.14$

2. Determine the independent and dependent quantities.

   The month number is the independent quantity, since the account balance depends on the month. The account balance is the dependent quantity.

SWB p. 133

3. Determine the common ratio that describes the change in the dependent quantity.

Organize your results in a table. Enter the independent quantity in the first column, and the dependent quantity in the second column. The balance at 0 months is the starting balance of the account, before the interest has been compounded. Because the independent quantity changes by one month at a time, analyzing the ratios between the dependent quantities will determine whether there is a common ratio between the dependent quantities.

Month	Account balance ($)	Ratio
0	5000	—
1	5015	$\frac{5015}{5000}=1.003$
2	5030.05	$\frac{5030.05}{5015}\approx 1.003$
3	5045.14	$\frac{5045.14}{5030.05}\approx 1.003$

The account balance has a common ratio: each term is 1.003 times the previous term.

4. Use the first pair of quantities and the common ratio to write an explicit function.

Because we have a common ratio, we will use an exponential model. The general form of an exponential function is $f(x) = a \bullet b^x + k$. In this case, a is the initial value (the value when $x = 0$), b is the common ratio, and k is 0 because the account does not have any amount removed from it. The function to represent the relationship between the number of months passed and the value of the account is $f(x) = 5000 \bullet 1.003^x$.

5. Evaluate the function to verify it is correct.

   Organize your results in a table. Use the explicit function to find each dependent term. The terms that are calculated should match the terms in the original list.

Month	Account balance ($)
0	$5000 \bullet 1.003^0 = 5000$
1	$5000 \bullet 1.003^1 = 5015$
2	$5000 \bullet 1.003^2 \approx 5030.05$
3	$5000 \bullet 1.003^3 \approx 5045.14$

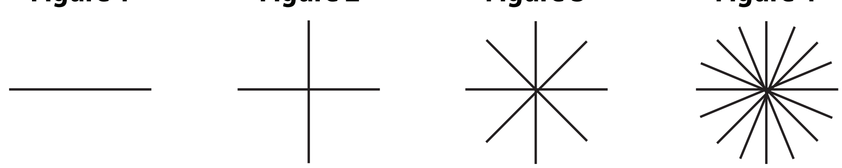
   The dependent terms match the ones in the original pattern, so the explicit function is correct.

   The relationship between the number of months passed and the account balance can be described using the function $f(x) = 5000 \bullet 1.003^x$.

Example 2

Consider that the first figure in the following diagram has two 180° angles, one on each side of the line segment. Each of these angles is then bisected or cut in half. This pattern continues, and the first 4 figures in the pattern are shown.

Figure 1 **Figure 2** **Figure 3** **Figure 4**



Write an explicit function to represent the relationship between the figure number and the number of angles in the figure.

1. Use the figures to determine the number of angles in figures 1, 2, 3, and 4.

   Count the angles in each figure, taking into consideration the note that the first figure has 2 angles.

   Figure 1: 2 angles

   Figure 2: 4 angles

   Figure 3: 8 angles

   Figure 4: 16 angles

2. Define the independent and dependent quantities.

   The figure number is the independent quantity, and the number of angles in the figure is the dependent quantity.

3. Determine if there is a common difference or a common ratio that describes the change in the dependent quantity.

   Organize your results in a table. Enter the independent quantity in the first column, and the dependent quantity in the second column. The pattern appears to have a common ratio. Use a table to find the ratio between the terms. Divide the current term by the previous term.

Figure	Number of angles	Ratio
1	2	
2	4	$\frac{4}{2}=2$
3	8	$\frac{8}{4}=2$
4	16	$\frac{16}{8}=2$

   The common ratio between the dependent terms is 2.

4. Use the first pair of quantities and the common ratio to write an explicit function.

   The common ratio is b, and a_1 is the value of the dependent quantity when the independent quantity is 1. The general equation to represent the relationship is $f(x) = a_1 b^{x-1}$. In this case, $a_1 = 2$ and $b = 2$, so the function to represent the relationship is $f(x) = 2 \bullet 2^{x-1}$.

5. Evaluate the function to verify that it is correct.

   Organize your results in a table. Use the explicit function to find each dependent term. The terms that are calculated should match the terms in the original list.

Figure	Number of angles
1	$2 \bullet 2^{1-1} = 2$
2	$2 \bullet 2^{2-1} = 4$
3	$2 \bullet 2^{3-1} = 8$
4	$2 \bullet 2^{4-1} = 16$

   The dependent terms match the ones in the original pattern, so the explicit function is correct.

   The relationship between the number of angles and the figure number can be described using the function $f(x) = 2 \bullet 2^{x-1}$.

Example 3

A population of bears in Alaska is declining by 7% each year. Currently, there are 237 bears. Find how many bears there will be after 1, 2, 3, or 4 years. Describe the number of bears after *x* years with an explicit function.

1. Use the description of the population decline to find how many bears there will be after 1, 2, 3, or 4 years.

   Currently, there are 237 bears. After 1 year, the population will decline by 7%. This means there will be $237 - 237 \bullet 0.07 = 220.41 \approx 220$ bears after 1 year.

   After 2 years, the population will decline from its year 1 population by another 7%. This means there will be $220.41 - 220.41 \bullet 0.07 \approx 204.98 \approx 205$ bears after 2 years.

   After 3 years, the population will decline from its year 2 population by another 7%. This means there will be $204.98 - 204.98 \bullet 0.07 \approx 190.63 \approx 191$ bears after 3 years.

   After 4 years, the population will decline from its year 3 population by another 7%. This means there will be $190.63 - 190.63 \bullet 0.07 \approx 177.29 \approx 177$ bears after 4 years.

2. Identify the independent and dependent quantities.

   The number of bears is dependent on the number of years passed, so the number of bears is the dependent quantity and the number of years passed is the independent quantity.

3. Determine the common ratio between the dependent terms.

   There is a common ratio between the dependent terms, and we can use a table to find it. To do so, divide the current term by the previous term.

Year	Number of bears	Ratio
0	237	—
1	220	$\frac{220}{237} \approx 0.93$
2	205	$\frac{205}{220} \approx 0.93$
3	191	$\frac{191}{205} \approx 0.93$
4	177	$\frac{177}{191} \approx 0.93$

   The common ration between the dependent terms is 0.93.

4. Use the common ratio to write an explicit function.

   Because we have a common ratio, we will use an exponential model. The general form of an exponential function is $f(x) = a \bullet b^x + k$. In this case, a is the initial population (the population when $x = 0$), b is the common ratio, and k is 0 because no constant value is removed from the bear population. The function to represent the relationship between the number of months passed and the value of the account is $f(x) = 237 \bullet 0.93^x$.

5. Evaluate the function to verify it is correct.

Organize your results in a table. Use the explicit function to find each term. The terms that are calculated should match the terms in the original list.

Year	Number of bears
0	$237 \bullet 0.93^0 = 237$
1	$237 \bullet 0.93^1 = 220.41 \approx 220$
2	$237 \bullet 0.93^2 \approx 204.98 \approx 205$
3	$237 \bullet 0.93^3 \approx 190.63 \approx 191$
4	$237 \bullet 0.93^4 \approx 177.29 \approx 177$

The pairs of independent and dependent quantities in the table match the ones in the original pattern, so the explicit function is correct.

The number of bears after any number of years, x, can be represented using the function $f(x) = 237 \bullet 0.93^x$.