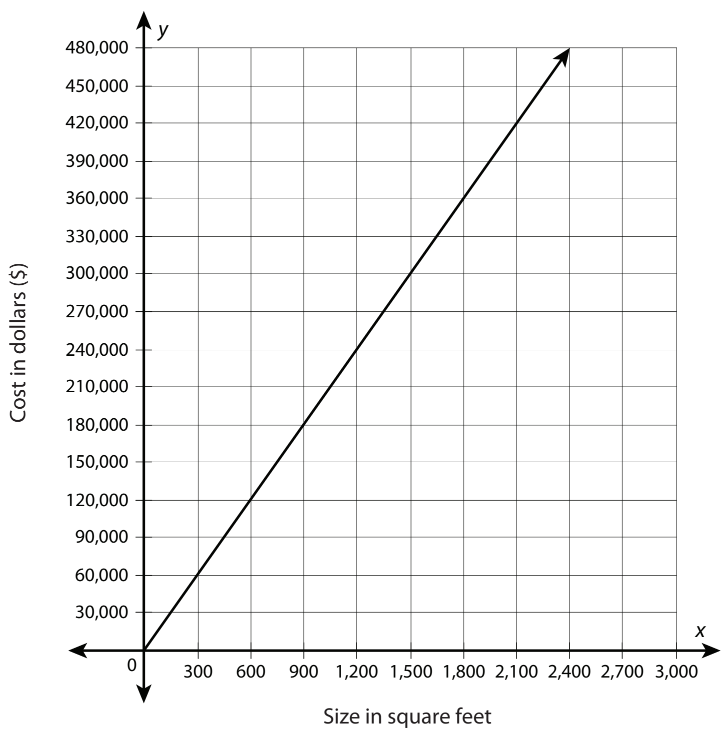
Guided Practice 6.3.1

Example 1

The following graph shows a linear model that approximates the relationship between the size of a home and how much it costs. The *x*-axis represents size in square feet, and the *y*-axis represents cost in dollars. Calculate the slope and the *y*-intercept of the linear model, and describe what these values mean in terms of housing prices.



1. Find the equation of the linear fit.

   The general equation of a line in slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.

   Find two points on the line using the graph.

   The graph contains the points (300, 60,000) and (600, 120,000).

   The formula to find the slope between two points (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

   Substitute (300, 60,000) and (600, 120,000) into the formula to find the slope.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$= \dfrac{120{,}000 - 60{,}000}{600 - 300}$	Substitute (300, 60,000) and (600, 120,000) for (x_1, y_1) and (x_2, y_2).
$= \dfrac{60{,}000}{300} = 200$	Divide.

   The slope between the two points (300, 60,000) and (600, 120,000) is 200.

   Find the y-intercept. Use the equation for slope-intercept form, $y = mx + b$, where b is the y-intercept.

   Replace x and y with values from a single point on the line. Let's use (300, 60,000).

   Replace m with the slope, 200. Solve for b.

$y = mx + b$	Equation for slope-intercept form
$(60{,}000) = (200)(300) + b$	Substitute values for x, y, and m.
$60{,}000 = 60{,}000 + b$	Multiply.
$0 = b$	Subtract 60,000 from both sides.

   The y-intercept of the linear model is 0.

   The equation of the line is $y = 200x$.

2. Determine the units of the slope.

   Divide the units on the y-axis by the units on the x-axis: $\frac{\text{dollars}}{\text{square feet}}$.

   The units of the slope are dollars per square foot.

3. Describe what the slope means in context.

   The slope is the change in cost of a home for each square foot of the home. The slope describes how price is affected by the size of the home purchased. A positive slope means the quantity represented by the y-axis increases when the quantity represented by the x-axis also increases.

   The cost of a home increases by $200 for each additional square foot of space in the home.

4. Determine the units of the y-intercept.

   The units of the y-intercept are the units of the y-axis: dollars.

5. Describe what the y-intercept means in context.

   The y-intercept is the value of the equation when $x = 0$, or when the size of the home is 0 square feet. For a home with no area, or for no home, the cost is $0.

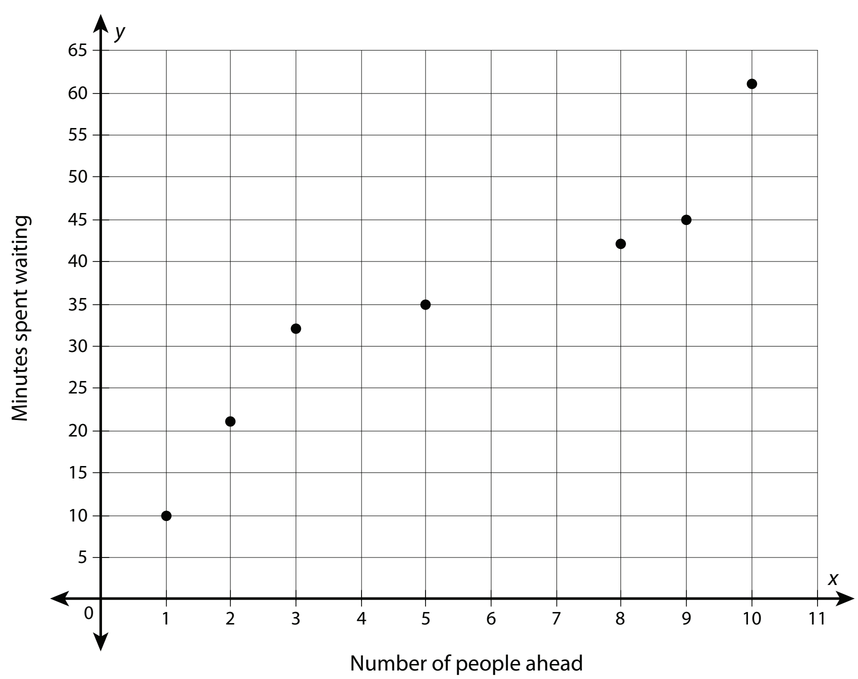
Example 2

A teller at a bank records the amount of time a customer waits in line and the number of people in line ahead of that customer when he or she entered the line. Describe the relationship between waiting time and the people ahead of a customer when the customer enters a line.

People ahead of customer	Minutes waiting
1	10
2	21
3	32
5	35
8	42
9	45
10	61

1. Create a scatter plot of the data.

   Let the *x*-axis represent the number of people ahead of the customer and the *y*-axis represent the minutes spent waiting.



2. Find the equation of a linear model to represent the data.

A line that fits the data well should have approximately the same number of data points both above and below the line. A line through the data points (2, 20) and (8, 45) appears to be a good approximation of the data. Find the slope.

The slope between two points (x_1, y_1) and (x_2, y_2) is $m=\dfrac{y_2-y_1}{x_2-x_1}$.

Substitute the points into the formula to find the slope.

$m=\dfrac{y_2-y_1}{x_2-x_1}$	Slope formula
$=\dfrac{45-20}{8-2}$	Substitute (2, 20) and (8, 45) for (x_1, y_1) and (x_2, y_2).
$=\dfrac{25}{6}\approx 4.2$	Divide.

The slope between the two points (2, 20) and (8, 45) is approximately 4.2.

Find the y-intercept. Use the equation for slope-intercept form, $y = mx + b$, where b is the y-intercept.

Replace x and y with values from a single point on the line. Let's use (2, 20).

Replace m with the slope, 4.2. Solve for b.

$y = mx + b$	Equation for slope-intercept form
$(20) = (4.2)(2) + b$	Substitute values for x, y, and m.
$20 = 8.4 + b$	Multiply.
$11.6 = b$	Subtract 8.4 from both sides.

The y-intercept of the linear model is approximately 11.6.

The equation of the line is $y = 4.2x + 11.6$.

3. Determine the units of the slope.

   Divide the units on the y-axis by the units on the x-axis:

   $$\frac{\text{minutes spent waiting}}{\text{number of people ahead}} = \frac{\text{minutes}}{\text{person}}$$

   The units of the slope are minutes per person.

4. Describe what the slope means in context.

   The slope describes how the waiting time increases for each person in line ahead of the customer. A customer waits approximately 4.2 minutes for each person who is in line ahead of the customer.

5. Determine the units of the y-intercept.

   The units of the y-intercept are the units of the y-axis: minutes.

6. Describe what the y-intercept means in context.

   The y-intercept is the value of y when $x = 0$, and represents the minutes spent waiting when the number of people ahead of the customer is 0. The y-intercept is 11.6.

   In this context, this y-intercept doesn't make sense, because a customer who enters the line with no people ahead of him or her would have a wait of 0 minutes, not 11.6 minutes. This discrepancy can be explained by closer analysis of the data. It appears that the first three data points on the graph follow a slope that appears to align with the origin, indicating a 0-minute wait time if there are 0 customers ahead. However, the wait times generally seem to increase at a slower rate if there are more than 3 people in line, so the wait time for each additional person in line is less. This may be explained by the addition of another teller or by the tellers picking up their pace to accommodate the long line of customers.

Example 3

For hair that is 12 inches or longer, a hair salon charges for haircuts based on hair length according to the equation $y = 5x + 35$, where x is the number of inches longer than 12 inches (hair length – 12) and y is the cost in dollars. Describe what the slope and y-intercept mean in context.

1. Determine the units of the slope.

   Divide the units of the dependent variable, y, by the units of the independent variable, x:

   $$\frac{\text{cost in dollars}}{\text{hair length greater than 12 inches}} = \frac{\text{dollars}}{\text{inch}}$$

2. Describe what the slope means in context.

   The units of the slope are dollars per inch. The slope describes how the cost of the haircut increases for each inch of hair length greater than 12 inches. The slope of 5 means that the cost of the haircut increases by \$5 for every inch of hair length over 12 inches.

3. Determine the units of the y-intercept.

   The units of the y-intercept are the units of the dependent variable, y: dollars.

4. Describe what the y-intercept means in context.

   The y-intercept is the value of the equation when $x = 0$, or when hair length is not greater than 12 inches. The y-intercept is the cost of a haircut when a customer's hair is no longer than 12 inches. A haircut is \$35 if a customer's hair isn't longer than 12 inches.