Key Concepts, *continued*

- A **radical expression** contains a square root, which can be shown using the radical symbol, $\sqrt{}$. The **square root** of a number *x* is a number that, when multiplied by itself, equals *x*. For example, 6 • 6 = 36, so 6 is a square root of 36.

- The number under the radical symbol is called the **radicand**.

Key Concepts, *continued*

- The square root of a number for which there is no rational root is an irrational number, such as $\sqrt{2}$.
- A **perfect square** is a number that is the square of a whole number. For example, 9 is a perfect square because $9 = 3^2$.
- A square root will be a rational number only if the radicand is a perfect square.
- A square root will be an irrational number only if the radicand is not a perfect square.

Key Concepts, *continued*

- An **irreducible radical** is a radical whose radicand contains no perfect square factors. In other words, the radical cannot be further reduced.
- To reduce a radical expression, factor out any perfect squares from the radicand, then use the product property of radicals, $\sqrt{m \bullet n} = \sqrt{m} \bullet \sqrt{n}$, to rewrite the expression as a product of the perfect squares and any remaining factors. Finally, evaluate the square roots of the perfect squares and simplify.

Key Concepts, *continued*

- Alternatively, rewrite the radicand in terms of its prime factorization, then use the product property to rewrite the expression as a product of factors with even powers and any remaining factors. Finally, evaluate the square roots of the factors with even powers and simplify.
- If there is a radical in the denominator of a simplified expression, rewrite it by multiplying the numerator and denominator by the radical in the denominator. This process is known as rationalizing the denominator.

Key Concepts, *continued*

- Two radical expressions containing the same irreducible radical are called like terms. For example, $2\sqrt{2}$ and $3\sqrt{2}$ are like terms because they both contain $\sqrt{2}$.
- A radical expression can be rewritten using the following properties.

Key Concepts, *continued*

Properties of Radicals

Property	Formula	Example
Addition property To add two like radical terms, add the coefficients.	$a\sqrt{m}+b\sqrt{m}=(a+b)\sqrt{m}$	$2\sqrt{2}+3\sqrt{2}=5\sqrt{2}$
Product property The square root of a product is equal to the product of the square roots of the factors.	$\sqrt{m \bullet n}=\sqrt{m}\bullet\sqrt{n}$	$\sqrt{6}=\sqrt{2 \bullet 3}=\sqrt{2}\bullet\sqrt{3}$

(*continued*)

Key Concepts, *continued*

Properties of Radicals

Property	Formula	Example
Quotient property The square root of a quotient is equal to the quotient of the square roots of the dividend and the divisor.	$\sqrt{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}},\ n \neq 0$	$\sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2} \bullet \sqrt{3}}{3} = \frac{\sqrt{6}}{3}$

(*continued*)

Key Concepts, *continued*

Properties of Radicals

Property	Formula	Example
Power reduction property The square root of a number raised to an even power is equal to the number raised to half the original power.	$\sqrt{n^{2a}} = n^a$	$\sqrt{16} = \sqrt{2^4} = 2^2 = 4$

(*continued*)

Key Concepts, *continued*

Properties of Radicals

Property	Formula	Example
Rational denominator property To rationalize the denominator, multiply the numerator and denominator by the radical in the denominator.	$\frac{m}{\sqrt{n}}=\frac{m}{\sqrt{n}}\bullet\frac{\sqrt{n}}{\sqrt{n}}=\frac{m\sqrt{n}}{n}$	$\frac{2}{\sqrt{2}}=\frac{2}{\sqrt{2}}\bullet\frac{\sqrt{2}}{\sqrt{2}}=\frac{2\sqrt{2}}{2}$

Common Errors/Misconceptions

- incorrectly categorizing a root as irrational when an exact root exists
- incorrectly evaluating a root
- multiplying only the denominator by the radical when rationalizing a denominator
- incorrectly identifying a repeating or terminating decimal as irrational

Guided Practice

Example 1

Reduce the radical expression $\sqrt{\frac{80}{5^4}}$. If the result has a root in the denominator, rationalize it. Is the result rational or irrational?

Guided Practice: Example 1, *continued*

1. Rewrite each number in the expression as a product of prime numbers.

The denominator of the expression under the radical sign, 5^4, is already written as a prime factorization. Rewrite the numerator, 80, as the product of its prime factors, then group identical factors together using exponents, as shown on the next slide.

Guided Practice: Example 1, *continued*

$\sqrt{\dfrac{80}{5^4}}$	Original expression
$\sqrt{\dfrac{2 \bullet 2 \bullet 2 \bullet 2 \bullet 5}{5^4}}$	Rewrite 80 as a product of its prime factors.
$\sqrt{\dfrac{2^4 \bullet 5}{5^4}}$	Group prime factors together with exponents.

Guided Practice: Example 1, *continued*

2. Cancel where possible to reduce the resulting expression.

Divide out factors that appear in both the numerator and the denominator.

The expression $\sqrt{\frac{2^4 \bullet 5}{5^4}}$ has 5s in the numerator and denominator, so 5 is a common factor that will cancel out.

Recall that when canceling terms in an expression that contains an exponent, you reduce the power of the exponent by 1 for each factor you cancel.

Guided Practice: Example 1, *continued*

$\sqrt{\dfrac{2^4 \bullet 5}{5^4}}$	Expression from the previous step
$\sqrt{\dfrac{2^4 \bullet \cancel{5}}{5^{\cancel{4}\,3}}}$	Cancel out common factors.
$\sqrt{\dfrac{2^4}{5^3}}$	Reduced expression

There are no more factors that appear in both the numerator and denominator, so nothing more can be cancelled out.

Guided Practice: Example 1, *continued*

3. Use the properties of radicals to rewrite the reduced expression.

Rewrite the expression as a fraction of radicals, and solve any squares under the radical sign.

Guided Practice: Example 1, *continued*

$\sqrt{\dfrac{2^4}{5^3}}$	Expression from the previous step
$\dfrac{\sqrt{2^4}}{\sqrt{5^3}}$	Rewrite using the quotient property of radicals.
$\dfrac{\sqrt{2^4}}{\sqrt{5^2 \bullet 5}}$	Factor out the perfect square in the denominator.

Guided Practice: Example 1, *continued*

$$\frac{\sqrt{2^4}}{\sqrt{5^2} \bullet \sqrt{5}}$$

Rewrite using the product property of radicals.

$$\frac{2^2}{5\sqrt{5}}$$

Evaluate the radicals.

$$\frac{4}{5\sqrt{5}}$$

Evaluate the exponent.

Guided Practice: Example 1, *continued*

4. **Rationalize the denominator of the resulting fraction.**

   To rationalize the denominator, multiply both the numerator and the denominator by the radical in the denominator. This is equivalent to multiplying by 1, and thus does not change the value of the fraction.

Guided Practice: Example 1, *continued*

$$\frac{4}{5\sqrt{5}}$$

Expression from the previous step

$$\frac{4}{5\sqrt{5}} \bullet \frac{\sqrt{5}}{\sqrt{5}}$$

Multiply the numerator and the denominator by the radical in the denominator.

$$\frac{4 \bullet \sqrt{5}}{5 \bullet 5}$$

Simplify.

$$\frac{4\sqrt{5}}{25}$$

Multiply.

Guided Practice: Example 1, *continued*

The expression $\sqrt{\frac{80}{5^4}}$ is equal to $\frac{4\sqrt{5}}{25}$.

Guided Practice: Example 1, *continued*

5. Determine whether the resulting expression is rational or irrational.

The expression $\frac{4\sqrt{5}}{25}$ cannot be written as a ratio of whole numbers and is therefore irrational.

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Guided Practice: Example 1, *continued*

Guided Practice

Example 2

Reduce the radical expression $\sqrt{16a^2} + \sqrt{32a^4}$.

Assuming *a* is a whole number, is the result rational or irrational?

Guided Practice: Example 2, *continued*

1. **Use the properties of radicals to rewrite the expression.**

   Rewrite each radical in the expression as a product of radicals, and evaluate where possible.

Guided Practice: Example 2, *continued*

$\sqrt{16a^2}+\sqrt{32a^4}$	Original expression
$\sqrt{16}\bullet\sqrt{a^2}+\sqrt{32}\bullet\sqrt{a^4}$	Rewrite using the product property of radicals.
$4\bullet a+\sqrt{32}\bullet a^2$	Evaluate the radical perfect squares, $\sqrt{16}$, $\sqrt{a^2}$, and $\sqrt{a^4}$.
$4a+\sqrt{32}\bullet a^2$	Simplify.

The radical expression $\sqrt{16a^2}+\sqrt{32a^4}$ can be rewritten as $4a+\sqrt{32}\bullet a^2$.

Guided Practice: Example 2, *continued*

2. Reduce any remaining radicals.

We have one remaining radical, $\sqrt{32}$. Rewrite 32 as a product with a perfect square, and simplify using the properties of radicals.

Guided Practice: Example 2, *continued*

$4a+\sqrt{32}\bullet a^2$	Simplified expression from the previous step
$4a+\sqrt{16\bullet 2}\bullet a^2$	Factor out the perfect square in the radicand.
$4a+\sqrt{16}\bullet\sqrt{2}\bullet a^2$	Rewrite using the product property of radicals.
$4a+4\bullet\sqrt{2}\bullet a^2$	Evaluate the radical perfect square, $\sqrt{16}$.
$4a+4\sqrt{2}\bullet a^2$	Simplify.

Guided Practice: Example 2, *continued*

The remaining radical, $\sqrt{2}$, cannot be further reduced. The final reduced expression is $4a + 4\sqrt{2} \bullet a^2$.

Guided Practice: Example 2, *continued*

3. **Determine whether the resulting expression is rational or irrational.**

   Because *a* is a whole number, the first part of the reduced expression, $4a$, is rational. The second part of the expression, $4\sqrt{2} \bullet a^2$, is a product of rational numbers and an irreducible radical. Therefore, the second part of the expression is irrational.

   Because the sum of a rational number and an irrational number is irrational, the entire expression must be irrational.

Guided Practice: Example 2, *continued*