Introduction

Measurement is a way of describing and quantifying the real world. A **quantity** is a number with meaning, so when you **quantify** something you are giving it meaning or measure. For example, if a gardener says, “This pumpkin weighs 10 pounds,” he is describing a property of the pumpkin (its heaviness) and quantifying the heaviness as “10 pounds.” Sometimes, the way a measurement or quantity is described can make it hard to solve a problem. For example, it’s hard to tell whether Mary will get to school on time if we are told she walks at a speed of 4,470 light years per aeon.

Introduction, *continued*

Or, you might be trying to compare two measurements given in different quantities. For instance, suppose you need to know which is larger: a liter or a quart. In such situations, you must convert the units of measurement into more practical terms.

Key Concepts

- A **unit of measurement** is a defined quantity of whatever is being measured. For example, the current formal definition of a meter is “the length of the path traveled by light in a vacuum during a time interval of $\frac{1}{299,792,458}$ of a second.” Another example of a unit of measure is “the width of my hand.”

Key Concepts, *continued*

- Every measurement has two parts: a number and a unit.
- The unit of a measurement tells you what is being measured, while the number of a measurement tells you how much of the unit there is. For example, the measurement “3 inches” tells you that you are measuring a length and that the length is 3 when measured in inches.
- The same measurement can have very different numerical parts depending on the units used. For example, a measurement of 2 miles is the same as 3,218.69 meters.

Key Concepts, *continued*

- **Standard units** are widely accepted units of measurement. Standard units are usually defined by law. For example, standard units in the United States include gallons, feet, and pounds.
- A **system of measurement** is a collection of units of measurements that all relate to each other. The metric system, or SI, is an example of a system of measurement.
- Two separate measurements generally need to be in the same unit before they can be compared.

Key Concepts, *continued*

- A **conversion factor** is a ratio of quantities given in different units that are equivalent. For example, the ratio $\frac{12 \text{ inches}}{1 \text{ foot}}$ is a conversion factor because 12 inches is the same length as 1 foot.

Key Concepts, *continued*

- To convert from one set of units to another, start with the quantity you want to convert, and multiply by a series of conversion factors that connect the beginning units with the ending units until you reach the desired units. The unwanted units should cancel with one another if the problem has been set up correctly.

Key Concepts, *continued*

- When setting up the conversion, unwanted units should appear in the numerator and the denominator an equal number of times, while the desired units should appear only once. It is also important to pay attention to the location of the units. Depending on what the target units are, the conversion factors and even the beginning unit ratio may need to be "flipped."

Key Concepts, *continued*

- For example, convert 10 meters per second to feet per second. When setting up the conversion, “feet” and “second” should each appear once. The unit “meters” is not wanted, so “meters” should appear in the numerator of the first ratio (comparing meters to seconds), and in the denominator of the second ratio (comparing feet to seconds). This leaves the unit “seconds” to be in the denominator of the first ratio, and the unit “feet” to be in the numerator of the second ratio.

Key Concepts, *continued*

- Therefore, the first ratio will be $\frac{10 \text{ meters}}{1 \text{ second}}$, because this places "meters" in the numerator and "seconds" in the denominator. Using the approximation 1 meter = 3.28 feet, we can write the second ratio, $\frac{3.28 \text{ feet}}{1 \text{ meter}}$. Finally, write the two ratios as a multiplication problem: $\frac{10 \text{ meters}}{1 \text{ second}} \cdot \frac{3.28 \text{ feet}}{1 \text{ meter}}$.

Key Concepts, *continued*

- To complete the conversion, cancel units appearing in the numerator and denominator, then
- multiply the number values:

$$\frac{10 \cancel{\text{ meters}}}{1 \text{ second}} \cdot \frac{3.28 \text{ feet}}{1 \cancel{\text{ meter}}}$$ Cancel units.

$$\frac{10 \cdot 3.28 \text{ feet}}{1 \text{ second}}$$ Multiply the number values.

$$\frac{32.8 \text{ feet}}{1 \text{ second}}$$ Converted measurement

Common Errors/Misconceptions

- using the wrong conversion factor(s)
- misidentifying the initial or target units
- incorrectly manipulating units with powers; for example, cancelling the whole unit expression instead of just one power: $\frac{1 \text{ meter}}{1 \text{ second}} \cdot \frac{1000 \text{ liters}}{1 \text{ meter}^3} \square \frac{1000 \text{ liters}}{1 \text{ second}}$

Guided Practice

Example 2

How many seconds are there in 1 day?

Guided Practice: Example 2, *continued*

1. Determine the target units.

We want to convert days to seconds. In other words, we need to know how many seconds there are per day. The proper units will be $\frac{\text{seconds}}{\text{day}}$.

Guided Practice: Example 2, *continued*

2. Determine necessary conversion factors.

To convert days to seconds, use your knowledge of time measurement to change the units. There are 60 seconds in a minute, 60 minutes in an hour, and 24 hours in a day. The conversion factors are therefore $\frac{60 \text{ seconds}}{1 \text{ minute}}$, $\frac{60 \text{ minutes}}{1 \text{ hour}}$, and $\frac{24 \text{ hours}}{1 \text{ day}}$.

Guided Practice: Example 2, *continued*

3. Multiply the conversion factors.

In the previous step, we found that we will use three conversion factors to change days to seconds. Set the conversion factors in a multiplication problem so the intermediate factor will cancel out unwanted units in the other two, then multiply the numbers. In this case, we just multiply all three factors together in order of increasing unit size.

Guided Practice: Example 2, *continued*

$$\frac{60 \text{ seconds}}{1 \text{ minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot \frac{24 \text{ hours}}{1 \text{ day}}$$

Set up the calculation.

$$\frac{60 \text{ seconds}}{1 \text{ } \cancel{\text{minute}}} \cdot \frac{60 \text{ } \cancel{\text{minutes}}}{1 \text{ } \cancel{\text{hour}}} \cdot \frac{24 \text{ } \cancel{\text{hours}}}{1 \text{ day}}$$

Cancel units that appear in the numerator and denominator.

$$\frac{60 \cdot 60 \cdot 24 \text{ seconds}}{1 \text{ day}}$$

Simplify.

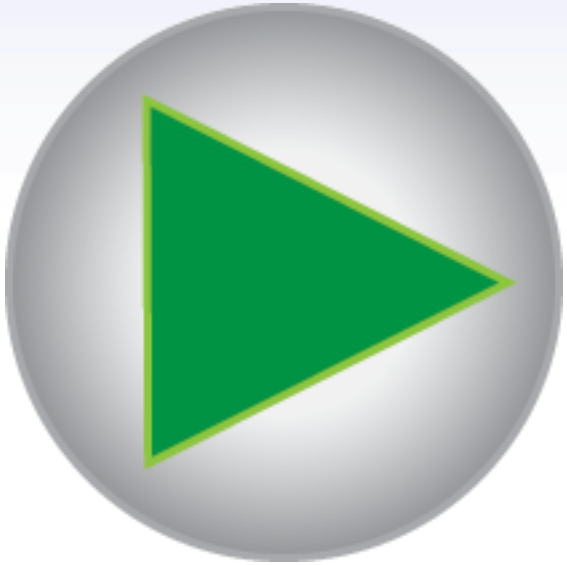
$$\frac{86{,}400 \text{ seconds}}{1 \text{ day}}$$

Multiply.

Guided Practice: Example 2, *continued*

There are 86,400 seconds in 1 day.

Guided Practice: Example 2, *continued*



Guided Practice

Example 3

Gemma is visiting friends in the U.S. She wants to make her famous mince pies, but her recipe lists most of the ingredients in grams. Use the chart to convert all the given measurements to U.S. units.

Ingredients:

225 grams cold butter, diced

340 grams plain flour

110 grams golden caster sugar

300 grams mincemeat

1 small egg

5 grams powdered sugar

Conversion Factors

U.S.	Metric
1 stick butter	113 grams
1 cup	225 grams
1 pound	455 grams
1 teaspoon	5 grams

Guided Practice: Example 3, *continued*

1. **Identify which units need to be converted.**

   All the ingredients except the egg are given in grams. Therefore, we will need to convert all the ingredients except the egg into U.S. measurements.

Guided Practice: Example 3, *continued*

2. Identify the target units.

In U.S. units, butter is usually measured in sticks, flour and sugar are usually measured in cups, meat is usually measured in pounds or ounces, and small amounts are usually measured in teaspoons. So, we want to convert butter to sticks, flour and caster sugar to cups, mincemeat to pounds, and powdered sugar to teaspoons.

Guided Practice: Example 3, *continued*

3. Set up the conversion and solve.

Use the list of conversion factors to set up the conversion. Because we are converting out of grams, the conversion factor should always have grams in the denominator and the target units in the numerator. The original units should be in terms of grams per recipe.

Guided Practice: Example 3, *continued*

For butter:

$$\frac{225 \text{ grams}}{1 \text{ recipe}} \cdot \frac{1 \text{ stick}}{113 \text{ grams}}$$

Set up the conversion.

$$\frac{225 \cancel{\text{ grams}}}{1 \text{ recipe}} \cdot \frac{1 \text{ stick}}{113 \cancel{\text{ grams}}}$$

Cancel units appearing in the numerator and denominator.

$$\frac{225 \text{ sticks}}{113 \text{ recipe}} \square \frac{2 \text{ sticks}}{1 \text{ recipe}}$$

Simplify.

This recipe calls for approximately 2 sticks of butter.

Guided Practice: Example 3, *continued*

For flour:

$$\frac{340 \text{ grams}}{1 \text{ recipe}} \cdot \frac{1 \text{ cup}}{225 \text{ grams}}$$

Set up the conversion.

$$\frac{340 \cancel{\text{ grams}}}{1 \text{ recipe}} \cdot \frac{1 \text{ cup}}{225 \cancel{\text{ grams}}}$$

Cancel units appearing in the numerator and denominator.

$$\frac{340 \text{ cups}}{225 \text{ recipes}} \square \frac{1.51 \text{ cups}}{1 \text{ recipe}}$$

Simplify.

This recipe calls for approximately $1\frac{1}{2}$ cups of flour.

Guided Practice: Example 3, *continued*

For sugar:

$$\frac{110 \text{ grams}}{1 \text{ recipe}} \cdot \frac{1 \text{ cup}}{225 \text{ grams}}$$

Set up the conversion.

$$\frac{110 \ \cancel{\text{grams}}}{1 \text{ recipe}} \cdot \frac{1 \text{ cup}}{225 \ \cancel{\text{grams}}}$$

Cancel units appearing in the numerator and denominator.

$$\frac{110 \text{ cups}}{225 \text{ recipes}} \square \frac{0.49 \text{ cups}}{1 \text{ recipe}}$$

Simplify.

This recipe calls for approximately $\frac{1}{2}$ cup of sugar.

Guided Practice: Example 3, *continued*

For mincemeat:

$$\frac{300 \text{ grams}}{1 \text{ recipe}} \cdot \frac{1 \text{ pound}}{455 \text{ grams}}$$

Set up the conversion.

$$\frac{300 \ \cancel{\text{grams}}}{1 \text{ recipe}} \cdot \frac{1 \text{ pound}}{455 \ \cancel{\text{grams}}}$$

Cancel units appearing in the numerator and denominator.

$$\frac{300 \text{ pounds}}{455 \text{ recipes}} \approx \frac{0.66 \text{ pounds}}{1 \text{ recipe}}$$

Simplify.

This recipe calls for approximately $\frac{2}{3}$ pound of mincemeat.

Guided Practice: Example 3, *continued*

For powdered sugar:

$$\frac{5 \text{ grams}}{1 \text{ recipe}} \cdot \frac{1 \text{ teaspoon}}{5 \text{ grams}}$$

Set up the conversion.

$$\frac{5 \ \cancel{\text{grams}}}{1 \text{ recipe}} \cdot \frac{1 \text{ teaspoon}}{5 \ \cancel{\text{grams}}}$$

Cancel units appearing in the numerator and denominator.

$$\frac{5 \text{ teaspoons}}{5 \text{ recipes}} = \frac{1 \text{ teaspoon}}{1 \text{ recipe}}$$

Simplify.

This recipe calls for 1 teaspoon of powdered sugar.

Guided Practice: Example 3, *continued*

4. Use the conversions to rewrite the list of ingredients.

The ingredients list, in U.S. units, is as follows:

2 sticks cold butter, diced

$1\frac{1}{2}$ cups plain flour

$\frac{1}{2}$ cup golden caster sugar

$\frac{2}{3}$ pound mincemeat

1 small egg

1 teaspoon powdered sugar

Guided Practice: Example 3, *continued*