Introduction

Creating equations from context is important since most real-world scenarios do not involve the equations being given. An **equation** is a mathematical sentence that uses an equal sign (=) to show that two quantities are equal. A **quantity** is something that can be compared because it has a numerical value. In this lesson, contexts will be given and equations must be created from them and then used to solve the problems. Since these problems are all in context, units are essential because without them, the numbers have no meaning.

Key Concepts

- A **linear equation** is an equation in which the highest power of any variable is 1. This lesson focuses on solving linear equations that have just one variable and that can be written in the form $ax + b = c$, where a, b, and c are numbers and x is the variable. Often, the most difficult task in turning a context into an equation is determining what the variable is and how to represent it in an equation.
- A variable is a letter used to represent an unknown value or a value which changes. Once the equation is determined, solving for the variable follows the steps for isolating a variable that were learned previously.

Key Concepts, *continued*

- The **solution** is the value that makes the equation true.
- In some cases, the solution must be converted into different units. Multiplying by a unit rate or a ratio can do this.
- A **rate** is a ratio that compares different kinds of measurements. A **unit rate** is a ratio of two measurements, the second of which is 1, such as miles per (1) gallon.
- Use units that make sense, such as when reporting time; for example, if the time is less than 1 hour, report the time in minutes.

Key Concepts, *continued*

- Make sure the units match when you begin the problem. For example, do not include a value that is measured in meters per second with a value that uses centimeters per second in the same equation. Convert one of these to match the other.
- Think about rounding and precision. The more significant digits you include, the more precise the number is.
- When using measurement in calculations, only report to the nearest decimal place of the least precise measurement.

Key Concepts, *continued*

Creating Equations from Context
1. Read the problem statement carefully. 2. Reread the scenario and make a list of the known quantities. 3. Read the statement again, identifying the unknown quantity or variable. 4. Create an equation from the known quantities and variable(s). 5. Solve the equation for the variable. 6. Interpret the solution of the equation in terms of the context of the problem and convert units if necessary.

Common Errors/Misconceptions

- attempting to solve the problem without first reading/understanding the problem statement
- incorrectly setting up the equation
- misidentifying the variable
- forgetting to convert to the correct units
- setting up the ratio inversely when converting units
- reporting too many or too few decimal places

Guided Practice

Example 3

Suppose two brothers who live 55 miles apart decide to have lunch together. To prevent either brother from driving the entire distance, they agree to leave their homes at the same time, drive toward each other, and meet somewhere along the route. The older brother drives cautiously at an average speed of 60 miles per hour. The younger brother drives faster, at an average speed of 70 mph, but still within the speed limit. How long will it take the brothers to meet each other?

Guided Practice: Example 3, *continued*

1. **Read the problem statement carefully.**

Guided Practice: Example 3, *continued*

2. Reread the scenario and make a list of the known quantities.

Problems involving “how fast,” “how far,” or “how long” can often be solved with the distance equation, $d = rt$, where d is distance, r is the rate of speed, and t is time.

From the scenario, we know each brother’s rate of speed, and the total distance they have to drive to meet each other. Make a list:

The older brother’s rate is 60 mph.

The younger brother’s rate is 70 mph.

The sum of their distances is 55 miles.

Guided Practice: Example 3, *continued*

3. **Read the statement again, identifying the unknown quantity or variable.**

   The scenario asks "how long" it will take, so the variable is time, t.

Guided Practice: Example 3, *continued*

4. Create an equation from the known quantities and variable(s).

Step 2 showed that the distance equation is $d = rt$ or $rt = d$. Together the brothers will travel a distance, d, of 55 miles.

(older brother's rate)(t) + (younger brother's rate)(t) = 55

The rate r of the older brother is 60 mph and the rate of the younger brother is 70 mph. Substitute these values into the expression.

Guided Practice: Example 3, *continued*

$60t + 70t = 55$

Create a table to see this another way.

	Rate (*r*)	Time (*t*)	Distance (*d*)
Older brother	60 mph	*t*	$d = 60t$
Younger brother	70 mph	*t*	$d = 60t$

Together, they traveled 55 miles, so add the distance equations based on each brother's rate.

$60t + 70t = 55$

Guided Practice: Example 3, *continued*

5. Solve the equation for the variable.

$60t + 70t = 55$	Equation from the previous step
$130t = 55$	Combine the like terms, 60*t* and 70*t*.
$\frac{130t}{130} = \frac{55}{130}$	Divide both sides by 130.
$t \approx 0.423$	Simplify.

Guided Practice: Example 3, *continued*

Note: The answer was rounded to the nearest thousandth. Although we do not need this precision, it is good practice to retain more digits than are needed until the last step to avoid rounding errors. When talking about meeting someone, it is highly unlikely that anyone would report a time that is broken down into decimals, which is why the next step will convert the units.

Guided Practice: Example 3, *continued*

6. Interpret the solution in the context of the problem and convert to the appropriate units if necessary.

In the equation, *t* represents hours. Automobile speeds in the United States are typically given in miles per hour (mph). Therefore, this unit of measurement is appropriate. However, typically portions of an hour are reported in minutes unless the time given is $\frac{1}{2}$ of an hour.

Guided Practice: Example 3, *continued*

Convert 0.423 hours to minutes using 60 minutes = 1 hour.

60 min = 1 hr

$$0.423\text{ hr} \bullet \frac{60\text{ min}}{1\text{ hr}}$$

$$0.423\,\cancel{\text{hr}} \bullet \frac{60\text{ min}}{1\,\cancel{\text{hr}}} \approx 25.38\text{ minutes}$$

Guided Practice: Example 3, *continued*

Here again, rarely would a person report that they are meeting someone in 25.38 minutes. In this case, there is a choice of rounding to either 25 or 26 minutes. Either answer makes sense.

The two brothers will meet each other in 25 or 26 minutes.

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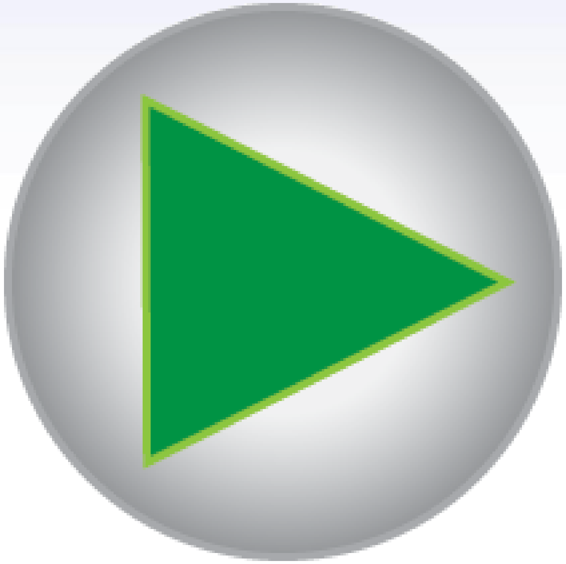
Guided Practice: Example 3, *continued*



Guided Practice

Example 5

Ernesto built a wooden car for a soap box derby. He is painting the top of the car blue and the sides black. He already has enough black paint, but needs to buy blue paint. He needs to know the approximate area of the top of the car to determine the size of the container of blue paint he should buy.

Guided Practice

Example 5

He measured the length to be 9 feet $11\frac{1}{4}$ inches, and the width to be $\frac{1}{2}$ inch less than 3 feet. What is the surface area of the top of the car? What is the most accurate area measurement Ernesto can use to buy his paint?

Guided Practice: Example 5, *continued*

1. **Read the problem statement carefully.**

Guided Practice: Example 5, *continued*

2. Reread the scenario and make a list of the known quantities.

The length is 9 feet 11.25 inches.

The width is 35.5 inches (3 feet = 36 inches;

$36 - \frac{1}{2}$ inch = 35.5 inches).

Guided Practice: Example 5, *continued*

3. **Read the statement again, identifying the unknown quantity or variable.**

   The scenario asks for the surface area of the car's top.

   Work with the accuracy component after calculating the surface area.

Guided Practice: Example 5, *continued*

4. Create an equation from the known quantities and variable(s).

The surface area will require some assumptions. A soap box derby car is tapered, meaning it is wider at one end than it is at another. To be sure Ernesto has enough paint, he assumes the car is rectangular with the width being measured at the widest location. Use the formula for area:

$$A = \text{length} \bullet \text{width} = lw$$

Guided Practice: Example 5, *continued*

For step 2, we listed the length and width, but they are not in the same units. The length is given in feet and inches; the width is given in inches.

Convert the length, 9 feet 11.25 inches, to inches.

$$9(12) + 11.25 = 119.25 \text{ inches}$$

The width is 35.5 inches.

Substitute the length and width into the formula $A = lw$.

$$A = lw$$

$$A = (119.25) \cdot (35.5)$$

Guided Practice: Example 5, *continued*

5. Solve the equation for the variable.

$A = 119.25 \cdot 35.5$

$A = 4233.375$

This gives a numerical result for the surface area, but the problem asks for the most accurate surface area measurement that can be calculated based on Ernesto's initial measurements. It is good practice to retain all three digits after the decimal place until the problem is complete to avoid inaccuracies introduced by rounding.

Guided Practice: Example 5, *continued*

6. Interpret the solution in the context of the problem and convert to the appropriate units if necessary.

In the equation, *A* represents area in square inches. When buying paint, the hardware store associate will ask how many square feet need to be covered. Ernesto has his answer in terms of square inches. Convert to square feet.

Guided Practice: Example 5, *continued*

There are 144 square inches in a square foot.

$$1\text{ ft}^2 = 144\text{ in}^2$$

$$4233.375\text{ in}^2 \bullet \frac{1\text{ ft}^2}{144\text{ in}^2}$$

$$4233.375\ \cancel{\text{in}^2} \bullet \frac{1\text{ ft}^2}{144\ \cancel{\text{in}^2}} = 29.3984375\text{ ft}^2$$

Guided Practice: Example 5, *continued*

7. Round the result to the appropriate number of places.

Ernesto's initial measurement with the fewest digits after the decimal is 35.5, which is reported to the nearest tenth. Thus, the final answer should also be rounded to the nearest tenth.

$$29.3984375 \text{ ft}^2 \approx 29.4 \text{ ft}^2$$

The surface area of the top of Ernesto's car is approximately 29.4 ft^2. This is the most accurate area measurement that Ernesto can use to buy his paint. ✔

Guided Practice: Example 5, *continued*