Paula's Peaches (Learning Task)

Name_____ Date_____

Mathematical Goals

- Factorization
- Solving quadratic equations

Essential Questions

- How do we use quadratic functions to represent contextual situations?
- How do we solve quadratic equations?
- How do we interpret quadratic functions in context?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

MGSE9-12.A.SSE.3a Factor any quadratic expression to reveal the zeros of the function defined by the expression

MGSE9-12.A.CED.2 Create ~~linear,~~ quadratic~~, and exponential~~ equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + {}^{r}/_{n})^{nt}$ has multiple variables.)

MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).

MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~.

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

MGSE9-12.F.IF.7a Graph ~~linear and~~ quadratic functions and show intercepts, maxima, and minima (as determined by the function or by context).

MGSE9-12.F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. *For example, compare and contrast quadratic functions in standard, vertex, and intercept forms.*

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

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Paula is a peach grower in central Georgia and wants to expand her peach orchard. In her current orchard, there are 30 trees per acre and the average yield per tree is 600 peaches. Data from the local agricultural experiment station indicates that if Paula plants more than 30 trees per acre, once the trees are in production, the average yield of 600 peaches per tree will decrease by 12 peaches for each tree over 30. She needs to decide how many trees to plant in the new section of the orchard.

1. Paula believes that algebra can help her determine the best plan for the new section of orchard and begins by developing a mathematical model of the relationship between the number of trees per acre and the average yield in **peaches per tree.**
   a. Is this relationship linear or nonlinear? Explain your reasoning.

   b. If Paula plants 6 more trees per acre, what will be the **average** yield in peaches per tree? What is the **average** yield in peaches per tree if she plants 42 trees per acre?

   c. Let T be the function for which the input x is the number of trees planted on each acre and $T(x)$ is the average yield in peaches per tree. Write a formula for $T(x)$ in terms of x and express it in simplest form. Explain how you know that your formula is correct.

   d. Draw a graph of the function T. Given that the information from the agricultural experiment station applies only to increasing the number of trees per acre, what is an appropriate domain for the function T?

2. Since her income from peaches depends on the total number of peaches she produces, Paula realized that she needed to take a next step and consider the total number of peaches that she can produce **per acre**.

   a. With the current 30 trees per acre, what is the yield in total peaches per acre? If Paula plants 36 trees per acre, what will be the yield in total peaches per acre? 42 trees per acre?

   b. Find the average rate of change of peaches per acre with respect to number of trees per acre when the number of trees per acre increases from 30 to 36. Write a sentence to explain what this number means.

   c. Find the average rate of change of peaches per acre with respect to the number of trees per acre when the number of trees per acre increases from 36 to 42. Write a sentence to explain the meaning of this number.

d. Is the relationship between number of trees per acre and yield in peaches per acre linear? Explain your reasoning.

e. Let *Y* be the function that expresses this relationship; that is, the function for which the input *x* is the number of trees planted on each acre and the output *Y*(*x*) is the total yield in peaches per acre. Write a formula for *Y*(*x*) in terms of *x* and express your answer in expanded form.

f. Calculate *Y*(30), *Y*(36), and *Y*(42). What is the meaning of these values? How are they related to your answers to parts a through c?

g. What is the relationship between the domain for the function *T* and the domain for the function *Y*? Explain.

3. Paula wants to know whether there is a different number of trees per acre that will give the same yield per acre as the yield when she plants 30 trees per acre.

a. Write an equation that expresses the requirement that *x* trees per acre yields the same total number of peaches per acre as planting 30 trees per acre.

b. Use the algebraic rules for creating equivalent equations to obtain an equivalent equation with an expression in *x* on one side of the equation and 0 on the other.

c. Multiply this equation by an appropriate rational number so that the new equation is of the form $x^2 + bx + c = 0$. Explain why this new equation has the same solution set as the equations from parts a and b.

d. When the equation is in the form $x^2 + bx + c = 0$, what are the values of *b* and *c*

e. Find integers *m* and *n* such that $m \cdot n = c$ and $m + n = b$.

f. Using the values of *m* and *n* found in part e, form the algebraic expression $(x + m)(x + n)$ and simplify it.

g. Combining parts d through f, rewrite the equation from part c in the form $(x + m)(x + n) = 0$.

h. This equation expresses the idea that the product of two numbers, $x + m$ and $x + n$, is equal to 0. We know from the discussion in Unit 2 that, when the product of two numbers is 0, one of the numbers has to be 0. This property is called the ***Zero Product Property***. For these particular values of *m* and *n*, what value of *x* makes $x + m = 0$ and what value of *x* makes $x + n = 0$?

i. Verify that the answers to part h are solutions to the equation written in part a. It is appropriate to use a calculator for the arithmetic.

j. Write a sentence to explain the meaning of your solutions to the equation in relation to planting peach trees.

4. Paula saw another peach grower, Sam, from a neighboring county at a farm equipment auction and began talking to him about the possibilities for the new section of her orchard. Sam was surprised to learn about the agricultural research and said that it probably explained the drop in yield for a orchard near him. This peach farm has more than 30 trees per acre and is getting an average total yield of 14,400 peaches per acre. *(Remember: Throughout this task assume that, for all peach growers in this area, the average yield is 600 peaches per tree when 30 trees per acre are planted and that this yield will decrease by 12 peaches per tree for each additional tree per acre.)*

   a. Write an equation that expresses the situation that x trees per acre results in a total yield per acre of 14,400 peaches per acre.

   b. Use the algebraic rules for creating equivalent equations to obtain an equivalent equation with an expression in x on one side of the equation and 0 on the other.

   c. Multiply this equation by an appropriate rational number so that the new equation is of the form $x^2 + bx + c = 0$. Explain why this new equation has the same solution set as the equations from parts and b.

   d. When the equation is in the form $x^2 + bx + c = 0$, what is value of b and what is the value of c?

   e. Find integers m and n such that $m \cdot n = c$ and $m + n = b$.

   f. Using the values of m and n found in part e, form the algebraic expression $(x + m)(x + n)$ and simplify it.

   g. Combining parts d through f, rewrite the equation from part d in the form $(x + m)(x + n) = 0$.

   h. This equation expresses the idea that the product of two numbers, $x + m$ and $x + n$, is equal to 0. We know from the discussion in Unit 2 that, when the product of two numbers is 0, one of the numbers has to be 0. What value of x makes $x + m = 0$? What value of x makes $x + n = 0$?

   i. Verify that the answers to part h are solutions to the equation written in part a. It is appropriate to use a calculator for the arithmetic.

j. Which of the solutions verified in part i is (are) in the domain of the function *Y*? How many peach trees per acre are planted at the peach orchard getting 14400 peaches per acre?

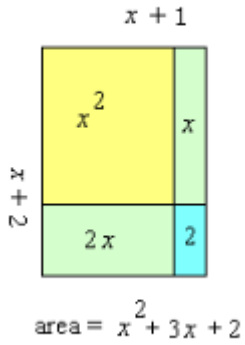
The steps in items 3 and 4 outline a method of solving equations of the form $x^2 + bx + c$. These equations are called ***quadratic equations*** and an expression of the form $x^2 + bx + c$ is called a ***quadratic expression***. In general, quadratic expressions may have any nonzero coefficient on the x^2 term. An important part of this method for solving quadratic expressions with coefficient 1 on the x^2term. An

important part of this method for solving quadratic equations is the process of rewriting an expression of the form $x^2 + bx + c$ in the form $(x + m)(x + n)$. The identity tells us that the product of the numbers m and n must equal c and that the sum of m and n must equal b.

5. Since the whole expression $(x + m)(x + n)$ is a product, we call the expressions $x + m$ and $x + n$ the **factors** of this product. For the following expressions in the form $x^2 + bx + c$, rewrite the expression as a product of factors of the form $x + m$ and $x + n$. Verify each answer by drawing a rectangle with sides of length $x + m$ and $x + n$, respectively, and showing geometrically that the area of the rectangle is $x^2 + bx + c$.

On a separate sheet of paper:

Example: $x^2 + 3x + 2$
Solution: (x + 1)*(x + 2)



a. $x^2 + 6x + 5$

b. $x^2 + 5x + 6$

c. $x^2 + 7x + 12$

d. $x^2 + 8x + 12$

e. $x^2 + 13x + 36$

f. $x^2 + 13x + 12$

6. In item 5, the values of *b* and *c* were positive. Now use Identity 1 in reverse to factor each of the following quadratic expressions of the form $x^2 + bx + c$ where *c* is positive but *b* is negative. Verify each answer by multiplying the factored form to obtain the original expression.

On a separate sheet of paper:

a. $x^2 - 8x + 7$
b. $x^2 - 9x + 18$
c. $x^2 - 4x + 4$
d. $x^2 - 8x + 15$
e. $x^2 - 11x + 24$
f. $x^2 - 11x + 18$
g. $x^2 - 12x + 27$

Paula's Peaches Continued!

7. Now we return to the peach growers in central Georgia. How many peach trees per acre would result in only 8400 peaches per acre? Which answer makes sense in this particular context?

8. If there are no peach trees on a property, then the yield is zero peaches per acre. Write an equation to express the idea that the yield is zero peaches per acre with *x* trees planted per acre, where *x* is number greater than 30. Is there a solution to this equation? Explain why only one of the solutions makes sense in this context.

9. At the same auction where Paula heard about the peach grower who was getting a low yield, she talked to the owner of a major farm supply store in the area. Paula began telling the store owner about her plans to expand her orchard, and the store owner responded by telling her about a local grower that gets 19,200 peaches per acre. Is this number of peaches per acre possible? If so, how many trees were planted?

10. Using graph paper, explore the graph of Y as a function of x.

    a. What points on the graph correspond to the answers for part j from questions 3 and 4?

    b. What points on the graph correspond to the answers to questions 7, 8, and 9?

    c. What is the relationship of the graph of the function Y to the graph of the function f, where the formula for $f(x)$ is the same as the formula for $Y(x)$ but the domain for f is all real numbers?

    d. Questions 4, 7, and 8 give information about points that are on the graph of f but not on the graph of Y. What points are these?

    e. Graph the functions f and Y on the same axes. How does your graph show that the domain of f is all real numbers? How is the domain of Y shown on your graph?

    f. Draw the line $y = 18000$ on the graph drawn for item 10, part e. This line is the graph of the function with constant value 18000. Where does this line intersect the graph of the function Y? Based on the graph, how many trees per acre give a yield of more than 18000 peaches per acre?

    g. Draw the line $y = 8400$ on your graph. Where does this line intersect the graph of the function Y? Based on the graph, how many trees per acre give a yield of <u>fewer than</u> 8400 peaches per acre?

    h. Use a graphing utility and this intersection method to find the number of trees per acre that give a total yield **closest** to the following numbers of peaches per acre:
    (i) 10000 (ii) 15000 (iii) 20000

    i. Find the value of the function Y for the number of trees given in answering (i) – (iii) in part c above.

11. In items 5 and 6, we used factoring as part of a process to solve equations that are equivalent to equations of the form $x^2 + bx + c = 0$ where *b* and *c* are integers. Look back at the steps you did in items 3 and 4, and describe the process for solving an equation of the form $x^2 + bx + c = 0$. Use this process to solve each of the following equations, that is, to find all of the numbers that satisfy the original equation. Verify your work by checking each solution in the original equation.

a. $x^2 - 6x + 8 = 0$

b. $x^2 - 15x + 36 = 0$

c. $x^2 + 28x + 27 = 0$

d. $x^2 - 3x - 10 = 0$

e. $x^2 + 2x - 15 = 0$

f. $x^2 - 4x - 21 = 0$

g. $x^2 - 7x = 0$

h. $x^2 + 13x = 0$

12. For each of the equations solved in question 11, do the following.

a. Use technology to graph a function whose formula is given by the left–hand side of the equation.
b. Find the points on the graph which correspond to the solutions found in question 8.
c. How is each of these results an example of the intersection method explored above?