Section 1-8 Radical Rules

PRODUCT RULE:

$\sqrt[a]{x} \cdot \sqrt[a]{y} = \sqrt[a]{xy}$

Example:

$\sqrt{10} \cdot \sqrt{x} = \sqrt{10x}$

QUOTIENT RULE:

$\frac{\sqrt[a]{x}}{\sqrt[a]{y}} = \sqrt[a]{\frac{x}{y}}$

Example:

$\frac{\sqrt{10}}{\sqrt{2}} = \sqrt{\frac{10}{2}} = \sqrt{5}$

More directly, when determining a product or quotient of radicals and the indices (the small number in front of the radical) are the same then you can rewrite 2 radicals as 1 or 1 radical as 2.

Simplify by rewriting the following using only one radical sign (i.e. rewriting 2 radicals as 1).

1. $\sqrt{3} \cdot \sqrt{12}$ $= \sqrt{3 \cdot 12}$
$= \sqrt{36}$
$= 6$

$\boxed{6}$

2. $\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$

$\boxed{2}$

3. $\sqrt{7x} \cdot \sqrt{2y}$

$\sqrt{7x \cdot 2y} = \sqrt{14xy}$

$\boxed{\sqrt{14xy}}$

4. $\frac{\sqrt[3]{12x^2}}{\sqrt[3]{4x}} = \sqrt[3]{\frac{12x^2}{4x}} = \sqrt[3]{\frac{12 \cdot \cancel{x} \cdot x}{4 \cdot \cancel{x}}}$
$= \sqrt[3]{3x}$

$\boxed{\sqrt[3]{3x}}$

Simplify by rewriting the following using multiple radical sign (i.e. rewriting 1 radical as 2).

5. $\sqrt{\frac{144}{25}} = \frac{\sqrt{144}}{\sqrt{25}} = \frac{12}{5}$

$\boxed{\frac{12}{5}}$

6. $\sqrt{\frac{x^6}{121}} = \frac{\sqrt{x^6}}{\sqrt{121}} = \frac{x^3}{11}$

$x^3 \cdot x^3 = x^6$

$\boxed{\frac{x^3}{11}}$

Express each radical in simplified form.

7. $\sqrt{48}$

$= \sqrt{4 \cdot 4 \cdot 3}$

$= \sqrt{4} \cdot \sqrt{4} \cdot \sqrt{3}$

$= 2 \cdot 2 \cdot \sqrt{3}$

Exact

$\boxed{4\sqrt{3}}$ P

8. $\sqrt{450x^4y^5}$

$3 \cdot 3 \cdot 5 \cdot 5 \cdot 2 \; x x x x \; y y y y y$

$\sqrt{9 \cdot 25 \cdot 2 \, x^2 \, x^2 \, y^2 y^2 \, y}$

$\sqrt{9} \cdot \sqrt{25} \cdot \sqrt{x^2} \cdot \sqrt{x^2} \cdot \sqrt{y^2} \cdot \sqrt{y^2} \cdot \sqrt{2y}$

$3 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot \sqrt{2y}$

$15x^2y^2\sqrt{2y}$

$\boxed{15x^2y^2\sqrt{2y}}$ N

9. $\sqrt{72x^5y^6}$

$3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \; x x x x x \; y y y y y y$

$\sqrt{9 \cdot 4 \cdot x^2 \cdot x^2 \cdot y^2 \cdot y^2 \cdot y^2 \cdot 2x}$

$\sqrt{9} \, \sqrt{4} \, \sqrt{x^2} \, \sqrt{x^2} \, \sqrt{y^2} \, \sqrt{y^2} \, \sqrt{y^2} \, \sqrt{2x}$

$3 \cdot 2 \cdot x \cdot x \cdot y \cdot y \cdot y \; \sqrt{2x}$

$\boxed{6x^2y^3\sqrt{2x}}$ O

10. $\sqrt{300x^{12}}$

$3 \; 2 \; 5 \; 5 \; 2 \; x x x x x x x x x x x x$

$2 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \sqrt{3}$

$\boxed{10x^6\sqrt{3}}$ O

11. $\sqrt{675x^4y^{11}}$

$5 \; 5 \; 3 \; 3 \; 3 \; x x x x \; y y y y y y y y y y y$

$5 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \; \sqrt{3y}$

$\boxed{15x^2y^5\sqrt{3y}}$ N

12. $-\sqrt{81x^3y^8}$

$3 \; 3 \; 3 \; 3 \; x x x \; y y y y y y y y$

$- \; 3 \cdot 3 \cdot x \cdot y \cdot y \cdot y \cdot y \; \sqrt{x}$

$\boxed{-9xy^4\sqrt{x}}$ A

13. $\sqrt[3]{48x^7y^3}$

$3 \; 2 \; 2 \; 2 \; 2 \; x x x x x x x \; y y y$

$\sqrt[3]{8} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{y^3} \cdot \sqrt[3]{6x}$

$2 \cdot x \cdot x \cdot y \; \sqrt[3]{6x}$

$\boxed{2x^2y\sqrt[3]{6x}}$ N

14. $\sqrt[3]{81x^{10}y^3}$

$3 \; 3 \; 3 \; 3 \; x x x x x x x x x x \; y y y$

$3 x x x y \cdot \sqrt[3]{3x}$

$\boxed{3x^3y\sqrt[3]{3x}}$ I

15. $\sqrt[3]{-27x^5}$

$-3 \cdot -3 \cdot -3 \; x x x x x$

$-3x \cdot \sqrt[3]{x^2}$

$\boxed{-3x\sqrt[3]{x^2}}$ K

Use the letters and answers to match the answer to the riddle. **Only some answers will be used.**

"What is an opinion without π?"

A	N	O	N	I	O	N
$-9xy^4\sqrt{x}$	$15x^2y^2\sqrt{2y}$	$10x^6\sqrt{3}$	$2x^2y\sqrt[3]{6x}$	$3x^3y\sqrt[3]{3x}$	$6x^2y^3\sqrt{2x}$	$15x^2y^5\sqrt{3y}$

Express each radical in simplified form.

16. $\sqrt{6x} \cdot \sqrt{12x} = \sqrt{6x \cdot 12x} = \sqrt{72x^2}$

$\sqrt{72x^2}$

$9 \cdot 8$

$3\ 3\ 4\ 2$

$3\ 3\ 2\ 2\ 2\ xx$

$3 \cdot 2 \cdot x\sqrt{2}$

$\boxed{6x\sqrt{2}}$

17. $\sqrt{18a^5} \cdot \sqrt{6a^4} = \sqrt{18a^5 \cdot 6a^4} = \sqrt{108a^9}$

$\sqrt{108a^9}$

$6\ \ 18$

$2 \cdot 3\ \ 3\ 6$

$2\ 3\ 3\ 3 \cdot 2\ aa\ aa\ aa\ aa\ a$

$2 \cdot 3 \cdot a \cdot a \cdot a \cdot a\sqrt{3a}$

$\boxed{6a^4\sqrt{3a}}$

Simplify. Assume that all variable represent positive real numbers.

18. $5\sqrt{3} + 1\sqrt{2} - 2\sqrt{3} + 4\sqrt{2}$

$3\sqrt{3} + 5\sqrt{2}$

$\boxed{3\sqrt{3} + 5\sqrt{2}}$

19. $\sqrt{108} + 5\sqrt{12} - 4\sqrt{44}$

$2\ 54$, $2\ 6 \cdot 9$, $2\ 2\ 3\ 3\ 3$; $4 \cdot 3$, $2\ 2\ 3$; $4\ 11$, $2\ 2\ 11$

$2 \cdot 3\sqrt{3} + 5 \cdot 2\sqrt{3} - 4 \cdot 2 \cdot \sqrt{11}$

$6\sqrt{3} + 10\sqrt{3} - 8\sqrt{11}$

$\boxed{16\sqrt{3} - 8\sqrt{11}}$

20. $2\sqrt{150} + \sqrt{18} + 3\sqrt{8} - \sqrt{24}$

$15 \cdot 10$, $3\ 5\ 5\ 2$; $9 \cdot 2$, $3\ 3\ 2$; $4\ 2$, $2\ 2\ 2$; $6\ 4$, $2\ 3\ 2\ 2$

$2 \cdot 5 \cdot \sqrt{6} + 3\sqrt{2} + 3 \cdot 2\sqrt{2} - 2\sqrt{6}$

$10\sqrt{6} + 3\sqrt{2} + 6\sqrt{2} - 2\sqrt{6}$

$\boxed{8\sqrt{6} + 9\sqrt{2}}$

21. $y\sqrt{18} - 3\sqrt{12y^4} + 2\sqrt{8y^2}$

$9\ 2$, $3\ 3\ 2$; $2 \cdot 6$, $2 \cdot 2\ 3\ yyyy$; $4\ 2$, $2\ 2\ 2\ yy$

$y \cdot 3 \cdot \sqrt{2} - 3 \cdot 2 \cdot y \cdot y\sqrt{3} + 2 \cdot 2 \cdot y\sqrt{2}$

$3y\sqrt{2} - 6y^2\sqrt{3} + 4y\sqrt{2}$

$\boxed{7y\sqrt{2} - 6y^2\sqrt{3}}$

22. $x\sqrt{32x^2} + 2\sqrt{18x^4}$

$8\ 4$, $2 \cdot 4\ 2\ 2$, $2\ 2\ 2\ 2\ 2\ xx$; $9\ 2$, $3\ 3\ 2\ xx\ xx$

$x \cdot 2 \cdot 2 \cdot x \cdot \sqrt{2} + 2 \cdot 3x \cdot x \cdot \sqrt{2}$

$4x^2\sqrt{2} + 6x^2\sqrt{2}$

$\boxed{10x^2\sqrt{2}}$

23. $5\sqrt{18x^4} - 3x\sqrt{8x^2} - x^2\sqrt{2}$

$9\ 2$, $3\ 3\ 2\ xx\ xx$; $4 \cdot 2$, $2\ 2\ 2\ xx$

$5 \cdot 3\ x\ x \cdot \sqrt{2} - 3 \cdot x \cdot 2 \cdot x \cdot \sqrt{2} - 1x^2\sqrt{2}$

$15x^2\sqrt{2} - 6x^2\sqrt{2} - 1x^2\sqrt{2}$

$\boxed{8x^2\sqrt{2}}$

24. $2\sqrt{10}\left(3\sqrt{6} + 2\sqrt{5} - \sqrt{24}\right)$

$6\sqrt{60} + 4\sqrt{50} - 2\sqrt{240}$

$6\ 10$, $3\ 2\ 2\ 5$; $5\ 10$, $5\ 5\ 2$; $24\ 10$, $6\ 4\ 2\ 5$, $3\ 2\ 2\ 2\ 2\ 5$

$6 \cdot 2 \cdot \sqrt{15} + 4 \cdot 5 \cdot \sqrt{2} - 2 \cdot 2 \cdot 2\sqrt{15}$

$12\sqrt{15} + 20\sqrt{2} - 8\sqrt{15}$

$\boxed{4\sqrt{15} + 20\sqrt{2}}$

25. $\sqrt{2x}\left(\sqrt{6x} + 3\sqrt{x}\right)$

$\sqrt{12x^2} + 3\sqrt{2x^2}$

$4\ 3$, $2\ 2\ 3\ xx$; $2\ xx$

$2 \cdot x \cdot \sqrt{3} + 3 \cdot x \cdot \sqrt{2}$

$\boxed{2x\sqrt{3} + 3x\sqrt{2}}$

26. $3\sqrt{6a}\left(\sqrt{4a} + 2\sqrt{15a^2}\right)$

$3\sqrt{24a^2} + 6\sqrt{90a^3}$

$4 \cdot 6$, $2 \cdot 2\ 2 \cdot 3\ aa$; $9\ 10$, $3\ 3\ 2\ 5\ aa\ a$

$3 \cdot 2a \cdot \sqrt{6} + 6 \cdot 3 \cdot a\sqrt{10a}$

$6a\sqrt{6} + 18a\sqrt{10a}$

$\boxed{6a\sqrt{6} + 18a\sqrt{10a}}$

Simplify. Assume that all variable represent positive real numbers and rationalize all denominators.

18. $\frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{\sqrt{25}} = \frac{3\sqrt{5}}{5}$

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3/√(5)
        1.341640786
3√(5)/5
        1.341640786
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$\boxed{\frac{3\sqrt{5}}{5}}$

19. $\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{9}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$

$\boxed{2\sqrt{3}}$

20. $\frac{3\sqrt{2}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{12}}{\sqrt{36}} = \frac{3\sqrt{12}}{6} = \frac{\sqrt{12}}{2} = \frac{\cancel{2}\sqrt{3}}{\cancel{2}}$

$\sqrt{12}$
$4 \cdot 3$
$2 \cdot 2 \quad 3$
$2\sqrt{3}$

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3√(2)/√(6)
        1.732050808
√(3)
        1.732050808
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$\boxed{\sqrt{3}}$

21. $\sqrt{\frac{16}{27}} = \frac{\sqrt{16}}{\sqrt{27}} = \frac{4}{3\sqrt{3}}$

$= \frac{4}{3\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3\sqrt{9}}$

$= \frac{4\sqrt{3}}{3 \cdot 3} = \boxed{\frac{4\sqrt{3}}{9}}$

$\boxed{\frac{4\sqrt{3}}{9}}$

22. $\frac{\left(\sqrt{12}+8\sqrt{3}-2\sqrt{27}\right)}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{\sqrt{24}+8\sqrt{6}-2\sqrt{54}}{2}$

$\frac{2\sqrt{6}+8\sqrt{6}-6\sqrt{6}}{2}$

$\frac{4\sqrt{6}}{2} = 2\sqrt{6}$

$\boxed{2\sqrt{6}}$

23. $\frac{\sqrt{2}\left(\sqrt{12}-\sqrt{3}\right)}{\sqrt{3}}$

$= \frac{\left(\sqrt{24}-\sqrt{6}\right)}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{72}-\sqrt{18}}{\sqrt{9}} = \frac{\sqrt{72}-\sqrt{18}}{3}$

$= \frac{6\sqrt{2}-3\sqrt{2}}{3}$

$= \frac{\cancel{3}\sqrt{2}}{\cancel{3}}$

$\sqrt{72}$
$9 \cdot 8$
$3\,3 \quad 4\,2$
$3\,3 \quad 2\,2\,2$
$3 \cdot 2 \cdot \sqrt{2}$

$\sqrt{18}$
$9\,2$
$3\,3\,2$

$\boxed{\sqrt{2}}$