

# Unit 3 – 3

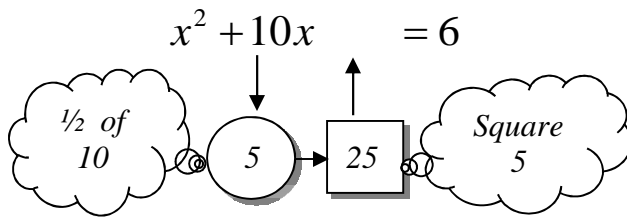
## BASIC

$$x^2 + 10x - 6 = 0$$

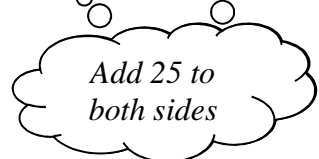
$$x^2 + 10x - 6 = 0$$

$$\begin{array}{r} +6 \\ +6 \end{array}$$

Move constant to the other side



$$x^2 + 10x + 25 = 6 + 25$$



$$x^2 + 10x + 25 = 31$$

Easily Factors

$$(x + 5)(x + 5) = 31$$

Can be re-written

$$(x + 5)^2 = 31$$

Take the square root of both sides

$$\sqrt{(x + 5)^2} = \sqrt{31}$$

Don't forget  $\pm$ .

$$x + 5 = \pm\sqrt{31}$$

$$\begin{array}{r} -5 \\ -5 \end{array}$$

Isolate x

$$x = -5 \pm \sqrt{31} \text{ OR } x \approx 0.5678 \text{ or } -10.5678$$

## INTERMEDIATE

$$3x^2 - 15x + 12 = 0$$

$$3x^2 - 15x + 9 = 0$$

$$\begin{array}{r} -9 \\ -9 \end{array}$$

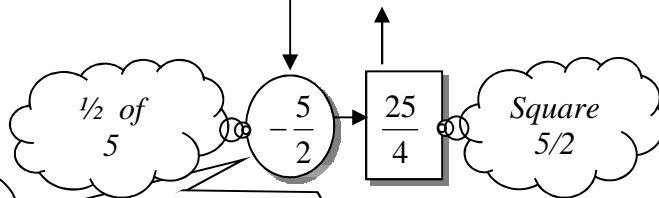
Move constant to the other side

$$3x^2 - 15x = -9$$

Divide both sides by the leading coefficients

$$\frac{3x^2 - 15x}{3} = \frac{-9}{3}$$

$$x^2 - 5x = -3 \longrightarrow x^2 + 10x + \frac{25}{4} = -\frac{3}{1} + \frac{25}{4}$$



Notice  $-\frac{5}{2}$  it is the number that when added to itself is equal to  $-5$  and when multiplied by itself is equal to  $\frac{25}{4}$ .

$$\left(x - \frac{5}{2}\right)\left(x - \frac{5}{2}\right) = -\frac{12}{4} + \frac{25}{4} \longrightarrow \left(x - \frac{5}{2}\right)^2 = \frac{13}{4}$$

$$\sqrt{\left(x - \frac{5}{2}\right)^2} = \pm\sqrt{\frac{13}{4}}$$

$$x - \frac{5}{2} = \pm\frac{\sqrt{13}}{2}$$

$$x = \frac{5}{2} \pm \frac{\sqrt{13}}{2}$$

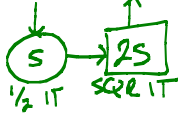
$$x = \frac{5 \pm \sqrt{13}}{2}$$

$$x \approx 0.6972 \text{ or } -4.3028$$

Solve the following by completing the square.

$$1. x^2 + 10x - 19 = 0$$

$$\underline{x^2 + 10x + 25 = 19 + 25}$$



$$(x+5)(x+5) = 44$$

$$\sqrt{(x+5)^2} = \pm\sqrt{44}$$

$$x+5 = \pm 2\sqrt{11}$$

$$\underline{x = -5 \pm 2\sqrt{11}}$$

$$x \approx -11.63 \text{ or } 1.63$$

$$\frac{25}{1, 25}$$

$$\frac{-1, -25}{5, 5}$$

$$\frac{-5, -5}{-5, -5}$$

$$\frac{(-11.63)^2 + 10(-11.63) - 19}{.63} = -19$$

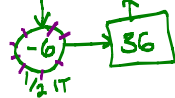
$$\frac{(1.63)^2 + 10(1.63) - 19}{.63} = -19$$

$$\frac{-5 + \sqrt{44}}{1} = 1.633249581$$

$$\frac{-5 - \sqrt{44}}{1} = -11.63324958$$

$$2. x^2 - 12x + 2 = 0$$

$$\underline{x^2 - 12x + 36 = -2 + 36}$$



$$(x-6)(x-6) = 34$$

$$\sqrt{(x-6)^2} = \pm\sqrt{34}$$

$$x-6 = \pm\sqrt{34}$$

$$\underline{x = 6 \pm \sqrt{34}} \leftarrow \text{EXACT}$$

$$x \approx 0.17 \text{ or } x \approx 11.83 \leftarrow \text{APPROXIMATE}$$

$$\frac{36}{-6, -6}$$

$$\sqrt{34}$$

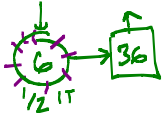
$$2 \sqrt{17}$$

$$\frac{6 + \sqrt{34}}{1} = 11.83095189$$

$$\frac{6 - \sqrt{34}}{1} = -1.690481052$$

$$3. x^2 + 12x - 8 = 0$$

$$\underline{x^2 + 12x + 36 = 8 + 36}$$



$$(x+6)(x+6) = 44$$

$$\sqrt{(x+6)^2} = \pm\sqrt{44}$$

$$x+6 = \pm 2\sqrt{11}$$

$$\underline{x = -6 \pm 2\sqrt{11}}$$

$$x \approx -12.63 \text{ or } x \approx 0.63$$

$$\frac{36}{6, 6}$$

$$\sqrt{44}$$

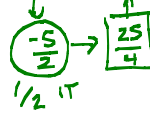
$$2\sqrt{11}$$

$$\frac{-6 + 2\sqrt{11}}{1} = 0.6332495807$$

$$\frac{-6 - 2\sqrt{11}}{1} = -12.63324958$$

$$4. x^2 - 5x - 4 = 0$$

$$\underline{x^2 - 5x + \frac{25}{4} = 4 + \frac{25}{4}}$$



$$(x - 5/2)(x - 5/2) = 41/4$$

$$\sqrt{(x - 5/2)^2} = \pm\sqrt{41/4}$$

$$x - 5/2 = \pm\sqrt{41}/2$$

$$\underline{x = \frac{5}{2} \pm \frac{\sqrt{41}}{2} = \frac{5 \pm \sqrt{41}}{2}} \leftarrow \text{EXACT}$$

$$x \approx -0.70 \text{ or } x \approx 5.70 \leftarrow \text{APPROX.}$$

$$4. \frac{4}{1} + \frac{25}{4}$$

$$\frac{16}{4} + \frac{25}{4} = \frac{41}{4}$$

$$\frac{(5 + \sqrt{41})/2}{1} = 5.701562119$$

$$\frac{(5 - \sqrt{41})/2}{1} = -0.7015621187$$

$$5. 2x^2 - 8x - 14 = 0$$

$$\underline{\frac{2x^2 - 8x}{2} = \frac{-14}{2}}$$

$$x^2 - 4x + 4 = -7 + 4$$



$$(x-2)(x-2) = 11$$

$$\sqrt{(x-2)^2} = \pm\sqrt{11}$$

$$x-2 = \pm\sqrt{11}$$

$$\text{EXACT} \rightarrow x = 2 \pm \sqrt{11}$$

$$\text{APPROX} \rightarrow x \approx -1.32 \text{ or } x \approx 5.32$$

$$\frac{4}{-2, -2}$$

$$\frac{2 + \sqrt{11}}{1} = 5.31662479$$

$$\frac{2 - \sqrt{11}}{1} = -1.31662479$$

$$6. 3x^2 - 18x - 5 = 0$$

$$\underline{\frac{3x^2 - 18x}{3} = \frac{-5}{3}}$$

$$x^2 - 6x + 9 = \frac{-5}{3} + 9$$



$$(x-3)(x-3) = \frac{32}{3}$$

$$\sqrt{(x-3)^2} = \pm\sqrt{\frac{32}{3}}$$

$$x-3 = \pm\sqrt{\frac{32}{3}}$$

$$x-3 = \pm\frac{4\sqrt{6}}{3}$$

$$\underline{x = 3 \pm \frac{4\sqrt{6}}{3}} \approx -0.27 \text{ or } 6.27$$

$$\sqrt{32}$$

$$4\sqrt{2}$$

$$\frac{4\sqrt{2} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{4\sqrt{6}}{3}$$

$$\frac{3 + 4\sqrt{6}/3}{1} = 6.265986324$$

$$\frac{3 - 4\sqrt{6}/3}{1} = -0.2659863237$$

Solve the following by completing the square.

$$7. 2x^2 - 16x - 3 = 0$$

$$\frac{2x^2 - 16x}{2} = \frac{3}{2}$$

$$x^2 - 8x + 16 = \frac{3}{2} + \frac{16 \cdot 2}{1 \cdot 2} \quad \frac{16}{-4, -4}$$

$$\begin{array}{c} \downarrow \quad \uparrow \\ \boxed{-4} \quad \boxed{16} \\ \downarrow \quad \uparrow \\ \frac{1}{2} \text{ IT} \end{array} = \frac{3}{2} + \frac{32}{2}$$

$$(x-4)(x-4) = \frac{35}{2}$$

$$\sqrt{(x-4)^2} = \pm \sqrt{\frac{35}{2}} \rightarrow \frac{\sqrt{35} \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{70}}{2}$$

$$x-4 = \pm \frac{\sqrt{70}}{2}$$

$$\frac{+4 \quad +4}{}$$

EXACT  $\rightarrow x = 4 \pm \frac{\sqrt{70}}{2}$  or  $\frac{8 \pm \sqrt{70}}{2}$

$$\begin{array}{l} 4 + \frac{\sqrt{70}}{2} \\ 8.183300133 \\ 4 - \frac{\sqrt{70}}{2} \\ -1.833001327 \end{array}$$

APPROX  $\rightarrow x \approx 8.18$  or  $x \approx -0.18$

$$8. 2x^2 - 10x - 6 = 0$$

$$\frac{2x^2 - 10x}{2} = \frac{6}{2}$$

$$x^2 - 5x + \frac{25}{4} = \frac{3}{2} + \frac{25}{4}$$

$$\begin{array}{c} \downarrow \quad \uparrow \\ \boxed{-\frac{5}{2}} \quad \boxed{\frac{25}{4}} \\ \downarrow \quad \uparrow \\ \frac{1}{2} \text{ IT} \end{array} = \frac{12}{4} + \frac{25}{4}$$

$$(x - \frac{5}{2})(x - \frac{5}{2}) = \frac{37}{4}$$

$$\sqrt{(x - \frac{5}{2})^2} = \pm \sqrt{\frac{37}{4}}$$

$$x - \frac{5}{2} = \pm \frac{\sqrt{37}}{2}$$

$$\frac{+5/2 \quad +5/2}{}$$

EXACT  $\rightarrow x = \frac{5}{2} \pm \frac{\sqrt{37}}{2} = \frac{5 \pm \sqrt{37}}{2}$

$$\frac{25/4}{-5/2, -5/2}$$

$$\begin{array}{l} (5 + \sqrt{37})/2 \\ 5.541381265 \\ (5 - \sqrt{37})/2 \\ -1.541381265 \end{array}$$

APPROX  $\rightarrow x \approx 5.54$  or  $x \approx -0.54$

$$9. 2x^2 - 5x - 12 = 0$$

$$\frac{2x^2 - 5x}{2} = \frac{12}{2}$$

$$x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{6}{1} + \frac{25}{16}$$

$$\frac{25}{16} \quad \frac{-5}{4}, \frac{-5}{4}$$

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$\begin{array}{c} \downarrow \quad \uparrow \\ \boxed{-\frac{5}{4}} \quad \boxed{\frac{25}{16}} \\ \downarrow \quad \uparrow \\ \frac{1}{2} \text{ IT} \quad \frac{5}{8} \text{ IT} \end{array} = \frac{96}{16} + \frac{25}{16}$$

$$(x - \frac{5}{4})(x - \frac{5}{4}) = \frac{121}{16}$$

$$\sqrt{(x - \frac{5}{4})^2} = \pm \sqrt{\frac{121}{16}}$$

$$x - \frac{5}{4} = \pm \frac{11}{4}$$

$$\frac{+5/4 \quad +5/4}{}$$

$$x = \frac{5}{4} \pm \frac{11}{4}$$

$$x = \frac{5}{4} + \frac{11}{4} \quad \text{or} \quad x = \frac{5}{4} - \frac{11}{4}$$

$$= \frac{16}{4} \quad \text{or} \quad x = \frac{-6}{4}$$

$$\boxed{x = 4 \quad \text{or} \quad x = -\frac{3}{2}} \leftarrow \text{EXACT}$$

$$10. 4x^2 - 10x - 8 = 0$$

$$\frac{4x^2 - 10x}{4} = \frac{8}{4}$$

$$x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{2}{1} + \frac{25}{16}$$

$$\frac{25}{16} \quad \frac{-5}{4}, \frac{-5}{4}$$

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$\begin{array}{c} \downarrow \quad \uparrow \\ \boxed{-\frac{5}{4}} \quad \boxed{\frac{25}{16}} \\ \downarrow \quad \uparrow \\ \frac{1}{2} \text{ IT} \quad \frac{5}{8} \text{ IT} \end{array} = \frac{32}{16} + \frac{25}{16}$$

$$(x - \frac{5}{4})(x - \frac{5}{4}) = \frac{57}{16}$$

$$\sqrt{(x - \frac{5}{4})^2} = \pm \sqrt{\frac{57}{16}}$$

$$x - \frac{5}{4} = \pm \frac{\sqrt{57}}{4}$$

$$\frac{+5/4 \quad +5/4}{}$$

EXACT  $\rightarrow x = \frac{5}{4} \pm \frac{\sqrt{57}}{4} = \frac{5 \pm \sqrt{57}}{4}$

APPROX  $\rightarrow x \approx 3.14$  or  $x \approx -0.64$

$$\begin{array}{l} (5 + \sqrt{57})/4 \\ 3.137458609 \\ (5 - \sqrt{57})/4 \\ -0.637458608 \end{array}$$