## UNIT $5 \cdot$ COMPARING AND CONTRASTING FUNCTIONS

## Answer Key

## Lesson 1: Key Features of Functions

Pre-Assessment, p. U5-1

1. c
2. d
3. c
4. c
5. b

## Warm-Up 5.1.1, p. U5-7

1. Answers may vary. The left end of the graph points down, which means the stock value was lower than $\$ 400$ before the 15 -week report began.
2. Answers may vary. The right end of the graph points up, which means the stock value is rising and may continue to increase over the coming weeks.
3. Answers will vary. Although there were fluctuations in the value of the stock over the 15 -week time period, the stock prices have risen overall and still seem to be increasing.

## Practice 5.1.1 A: Comparing Key Features of

## Different Functions, p. U5-21

1. left: approaching $-\infty$ right: approaching $-\infty$ quadratic
2. left: approaching -4 right: approaching $\infty$ exponential
3. left: approaching $\infty$ right: approaching $-\infty$ linear
4. left: approaching 0
right: approaching $\infty$ exponential
5. left: approaching $-\infty$ right: approaching $\infty$ linear
6. left: approaching $\infty$ right: approaching $\infty$ quadratic
7. left: approaching $\infty$ right: approaching 0 exponential
8. left: approaching $\infty$ right: approaching $\infty$ quadratic
9. left: approaching $\infty$ right: approaching $-\infty$ linear
10. Answers may vary. The end behavior tells you whether your balance is increasing or decreasing, which is important if you want to know how much money you have. Other key features: the $x$-intercept is when there is no money left in the account, and the $y$-intercept is the beginning balance (which is positive when there is money in the account and negative when the account is overdrawn).

## Practice 5.1.1 B: Comparing Key Features of Different Functions, p. U5-24

1. left: approaching $\infty$
right: approaching $\infty$ quadratic
2. left: approaching 6
right: approaching $-\infty$ exponential
3. left: approaching $-\infty$ right: approaching $\infty$ linear
4. left: approaching $\infty$ right: approaching $-\infty$ linear
5. left: approaching 1
right: approaching $\infty$ exponential
6. left: approaching $-\infty$ right: approaching $-\infty$ quadratic
7. left: approaching -2
right: approaching $\infty$ exponential
8. left: approaching $\infty$ right: approaching $\infty$ quadratic
9. left: approaching $\infty$ right: approaching $-\infty$ linear
10. decreasing; yes, $x \geq 0$

Warm-Up 5.1.2, p. U5-27

1. Answers may vary. The $x$-intercept is around $(13.3,0)$ and shows at how many months Amanda will run out of money.
2. The $y$-intercept is $(0,200)$ and shows the current time when Amanda has $\$ 200$ to spend.
3. Yes. According to the graph, she will be able to keep it for 13 months.

Practice 5.1.2 A: Graphing Different Functions Using Key Features, p. U5-42

1. exponential

2. linear

3. quadratic

4. exponential

5. not enough information given to specify only one type of function
6. linear

7. not enough information given to specify only one type of function
8. linear

9. exponential

10. quadratic


Practice 5.1.2 B: Graphing Different Functions Using Key Features, p. U5-44

1. quadratic

2. linear

3. exponential

4. not enough information given to specify only one type of function
5. quadratic

6. not enough information given to specify only one type of function
7. exponential

8. exponential

9. quadratic

10. linear


Progress Assessment, p. U5-46

1. b
2. c
3. c
4. a
5. d
6. d
7. b
8. a
9. d
10. a
11. Answers may vary. Linear and exponential functions both share the key feature of only one $x$-intercept, while all three types of functions can have only one $y$-intercept. Without additional key features, there is no way of knowing which function type it is.

## Lesson 2: Average Rate of Change

Pre-Assessment, p. U5-50

1. a
2. d
3. b
4. c
5. c

Warm-Up 5.2.1, p. U5-54

1. $\$ 105$
2. The cost, $c$, is dependent on the number of messages sent, $m$.
3. For each message sent, the total cost increases by $\$ 0.04$.

Practice 5.2.1 A: Patterns of Change for Different Functions, p. U5-70

1. linear
2. linear
3. exponential
4. exponential
5. quadratic
6. exponential
7. exponential
8. quadratic
9. quadratic
10. linear

Practice 5.2.1 B: Patterns of Change for Different
Functions, p. U5-72

1. exponential
2. linear
3. quadratic
4. exponential
5. exponential
6. linear
7. quadratic
8. linear
9. exponential
10. It is impossible to tell due to the gap in input values.

Warm-Up 5.2.2, p. U5-74

1. 200 mph
2. 20 hours
3. the A Train

Practice 5.2.2 A: Average Rate of Change on a Graph, p. U5-91

1. linear
2. exponential
3. exponential
4. quadratic
5. 4
6. -5
7. 70 mph
8. 40 mph
9. 5 mph
10. The ambulance will arrive first; its function has a fast enough rate of change to exceed the other two functions before its output reaches 25 .

## Practice 5.2.2 B: Average Rate of Change on a Graph, p. U5-97

1. quadratic
2. linear
3. exponential
4. exponential
5. 3
6. $-\frac{15}{4}$
7. 60 mph
8. 6 mph
9. 36 mph
10. The ambulance; it's hard to tell from the graph, but if you analyze the rates of change you will see that the police car function isn't able to outgrow the ambulance function before its output is 20 .

## Warm-Up 5.2.3, p. U5-101

1. for $f(x): 3$
for $g(x): 2$
for $h(x)$ : 3
2. The point $(1,3)$ represents a point where all three functions are equal.

Practice 5.2.3 A: Comparing Functions Using Average Rate of Change, p. U5-119

1. quadratic; positive
2. exponential; positive
3. linear; negative
4. as $x \rightarrow \infty, f(x) \rightarrow-\infty$ and as $x \rightarrow-\infty, f(x) \rightarrow 0$
5. as $x \rightarrow \infty, f(x) \rightarrow \infty$ and as $x \rightarrow-\infty, f(x) \rightarrow \infty$
6. as $x \rightarrow \infty, f(x) \rightarrow-\infty$ and as $x \rightarrow-\infty, f(x) \rightarrow \infty$
7. as $x \rightarrow \infty, f(x) \rightarrow \infty$ and as $x \rightarrow-\infty, f(x) \rightarrow 0$
8. as $x \rightarrow \infty, f(x) \rightarrow-\infty$ and as $x \rightarrow-\infty, f(x) \rightarrow-\infty$
9. as $x \rightarrow \infty, f(x) \rightarrow \infty$ and as $x \rightarrow-\infty, f(x) \rightarrow \infty$
10. No, $h(x)$ is an exponential function so its rate of change increases much more rapidly than the linear or quadratic function.

## Practice 5.2.3 B: Comparing Functions Using Average Rate of Change, p. U5-121

1. quadratic; negative
2. linear; positive
3. exponential; negative
4. as $x \rightarrow \infty, f(x) \rightarrow \infty$ and as $x \rightarrow-\infty, f(x) \rightarrow 0$
5. as $x \rightarrow \infty, f(x) \rightarrow-\infty$ and as $x \rightarrow-\infty, f(x) \rightarrow-\infty$
6. as $x \rightarrow \infty, f(x) \rightarrow-\infty$ and as $x \rightarrow-\infty, f(x) \rightarrow \infty$
7. as $x \rightarrow \infty, f(x) \rightarrow 0$ and as $x \rightarrow-\infty, f(x) \rightarrow \infty$
8. as $x \rightarrow \infty, f(x) \rightarrow \infty$ and as $x \rightarrow-\infty, f(x) \rightarrow \infty$
9. as $x \rightarrow \infty, f(x) \rightarrow \infty$ and as $x \rightarrow-\infty, f(x) \rightarrow \infty$
10. No, $h(x)$ is an exponential function so its rate of change increases much more rapidly than the linear or quadratic function.

## Progress Assessment, p. U5-123

1. b
2. c
3. a
4. a
5. d
6. d
7. d
8. b
9. a
10. b
11. a. 0.8
b. 1
c. 5 hours
d. The pH level in Tank 2 will greatly exceed the level in Tank 1.

## Lesson 3: Function Transformations

## Pre-Assessment, p. U5-128

1. b
2. c
3. a
4. b
5. d

## Warm-Up 5.3.1, p. U5-132

1. vertical translation
2. The $k$ of the translation should be the width of the stripe.

Practice 5.3.1 A: Translation, p. U5-145

1. $g(x)=2 x-6$
2. $g(x)=(x-1)^{2}$
3. $g(x)=7 x+3$
4. $g(x)=-\left(4^{x}\right)-7$
5. $g(x)=3(x+3)^{2}$
6. $g(x)=2^{x-2}$
7. $g(x)=-\left(x^{2}\right)+1$
8. $g(x)=-\left(7^{x}\right)+9$
9. $g(x)=\left(x^{2}\right)-6$
10. a

Practice 5.3.1 B: Translation, p. U5-148

1. $f(x)=3(x+5)+5=3 x+20 \quad$ 6. $g(x)=x^{2}-1$
2. $g(x)=5-3 x$
3. $f(x)=5^{x}+2$
4. $f(x)=4 x^{2}-3$
5. $g(x)=2^{x+2}-3$
6. $f(x)=2(x-1)^{2}$
7. $g(x)=-\left(2^{x}\right)-1$
8. $g(x)=-(x+2)^{2}$
9. b

## Warm-Up 5.3.2, p. U5-151

1. The transformation to be applied is a compression.
2. The dilation factor is the quantifier. The quantifier will be reapplied for each row to complete the perspective effect: the farther away, the smaller the wave.
3. The closer the stripes are to the top of the flag, the smaller the dilation factor of the original wave function.

Practice 5.3.2 A: Dilation and Reflection, p. U5-165

1. $g(x)=-3 x-1$
2. $g(x)=2 f(-x), k=2$
3. $g(x)=f\left(\frac{x}{2}\right), k=\frac{1}{2}$
4. $g(x)=-\left(3^{x}\right)-1$
5. $g(x)=-2+4 x^{2}$
6. $g(x)=2^{\frac{2 x}{5}}$
7. $g(x)=\frac{3}{2} x^{2}$
8. $g(x)=5^{-x}+1$
9. c
10. $g(x)=-\frac{1}{4} 3^{x}$

Practice 5.3.2 B: Dilation and Reflection, p. U5-169

1. $f(x)=-5+x$
2. $f(x)=g\left(\frac{x}{3}\right) ; k=\frac{1}{3}$
3. $f(x)=g(3 x), k=3$
4. $f(x)=4^{-x}-2$
5. $f(x)=-\left(x^{2}\right)-2$
6. $f(x)=3\left(2^{x}\right)$
7. $f(x)=\frac{3}{8} x^{2}$
8. $f(x)=7^{x}+2$
9. c
10. $f(x)=2\left(4^{x}\right)$

## Warm-Up 5.3.3, p. U5-173

1. She should use the $y$-axis.
2. She should reflect first across the $y$-axis and then the $x$-axis, or the $x$-axis first and then the $y$-axis.
3. She could apply a translation if she wants the shape to stay the same, or she could apply a dilation if she wants it to stretch in one or both directions.

Practice 5.3.3 A: Even and Odd Functions, p. U5-185

1. even
2. even
3. odd
4. odd
5. odd
6. neither even nor odd
7. neither even nor odd
8. even
9. d
10. neither even nor odd

Practice 5.3.3 B: Even and Odd Functions, p. U5-188

1. odd
2. odd
3. even
4. even
5. even
6. neither even nor odd
7. even
8. even
9. c
10. even

## Progress Assessment, p. U5-191

| 1. a | 6. b |
| :--- | ---: |
| 2. c | 7. c |
| 3. d | 8. d |
| 4. a | $9 . \mathrm{c}$ |
| 5. a | $10 . \mathrm{b}$ |

11. a. $g(x)=3 x^{2}-2$
b. $g(x)=\frac{3}{2} x^{2}$
c. $g(x)=-3 x^{2}$
d. All four functions are even.

## Lesson 4: Modeling with Functions

## Pre-Assessment, p. U5-195

1. a
2. b
3. d
4. b
5. a

Warm-Up 5.4.1, p. U5-199

1. $\$ 10$
2. $\$ 22$
3. $y=3 x+10$; linear
4. 3 units

Practice 5.4.1 A: Choosing a Model, p. U5-210

1. linear
2. $n^{2}$
3. $\$ 45$; linear
4. 4
5. $y=3 x+1$
6. 5 grams; exponential
7. 6
8. $y=3 x-1$
9. exponential
10. quadratic

Practice 5.4.1 B: Choosing a Model, p. U5-212

1. There are 2 fewer people in hour 2 than hour 1 ; linear
2. $\$ 19$
3. $y=-x+2$; linear
4. quadratic
5. quadratic
6. $2 n$
7. 49
8. 30 grams; exponential
9. $y=-5 x+2$
10. exponential

## Warm-Up 5.4.2, p. U5-214

1. $\$ 30$
2. $\$ 0$
3. $\$ 605$

Practice 5.4.2 A: Using a Model in Context, p. U5-228

1. $y=10 x+15 ; \$ 25, \$ 55$; no
2. $y=5 x^{2}$; 500 square feet
3. $y=10000(1.20)^{\frac{x}{3}} ; 17,280$; yes, because you can't have a fraction of an organism
4. $y=1.4 x ; \$ 2.80 ; \$ 8.40$
5. $y=136\left(\frac{1}{2}\right)^{\frac{x}{2}} ; 17$ grams
6. $y=3000(1.15)^{x} ; \$ 4,562.63$
7. $y=200\left(1+\frac{0.08}{4}\right)^{4 x} ; \$ 297.19$
8. $y=500\left(1+\frac{0.10}{12}\right)^{12 x} ; \$ 744.68$
9. $y=10000(1.13)^{x} ; \$ 7,569$
10. $y=750(1.2)^{x} ; 4$ years

Practice 5.4.2 B: Using a Model in Context, p. U5-230

1. $y=2 x+5 ; \$ 7 ; \$ 15$; no
2. $y=5 x ; \$ 25$
3. $y=5000(1.5)^{\frac{4}{4}} ; 25,313$ bacteria; because you can't have a
fraction of an organism
4. $y=3 x ; \$ 30$
5. $y=400\left(\frac{1}{2}\right)^{\frac{x}{8}} ; 25$ grams; no, because the amount gets divided in half each time, not subtracted; the function will approach 0 , but never cross it
6. $y=1500(1.05)^{x} ; \$ 1,653.75$
7. $y=7000\left(1+\frac{0.11}{12}\right)^{12 x} ; \$ 9,722.15$
8. $y=3500\left(1+\frac{0.09}{12}\right)^{12 x} ; \$ 4,580.26$
9. $y=30000(2)^{x} ; 1,875$ ants
10. $y=1300\left(1+\frac{0.07}{4}\right)^{4 x} ; \$ 2,113.04$

## Warm-Up 5.4.3, p. U5-232

1. The graph has a greater slope.
2. The function of the graph is greater at $x=3$.

Practice 5.4.3 A: Comparing Models, p. U5-246

1. graph B
2. After 5 cups, she earns more using plan A. After 10 cups, she earns the same amount for either plan.
3. \$4; more expensive
4. a
5. the function in the table
6. $g(x)$ is greater at $x=3$
7. the function in the table
8. the function equation
9. the function in the graph
10. $f(x)=\mathrm{B}, g(x)=\mathrm{A}$

## Practice 5.4.3 B: Comparing Models, p. U5-250

1. graph A
2. the first option
3. the first option
4. c
5. the function in the table
6. $f(x)$ is greater at $x=2$
7. the function in the graph
8. the function equation
9. the function equation
10. $f(x)=\mathrm{A}, g(x)=\mathrm{B}$

Progress Assessment, p. U5-254

1. c
2. d
3. a
4. a
5. a
6. b
7. c
8. a
9. b
10. b
11. a. 12
b. 2
c. Dana
d. yes, $f(x)$ has a horizontal shift of $-1 ; g(x)$ has a vertical shift of +1

## Unit Assessment

## p. U5-258

1. b
2. b
3. d
4. d
5. d
6. b
7. a
8. b
9. b
10. c
11. b
12. b
13. a. All three functions are defined on the domain $[0, \infty)$.
b. $A(x) \rightarrow 500$ as $x \rightarrow 0 ; A(x) \rightarrow \infty$ as $x \rightarrow \infty$.
c. $B(x) \rightarrow 525$ as $x \rightarrow 0 ; B(x) \rightarrow \infty$ as $x \rightarrow \infty$.
d. $C(x) \rightarrow 600$ as $x \rightarrow 0 ; C(x) \rightarrow \infty$ as $x \rightarrow \infty$.
e. As $x \rightarrow \infty$, the function $c(x)$ will eventually outgrow the other two functions. This is because $C(x)$ is an exponential function, and exponential functions always eventually grow faster than linear and quadratic functions.
14. a. $g(x)=4 x$
b. $h(x)=2 x+2.5$
c. $j(x)=-4 x+5$
d. $f(x)$ is neither even nor odd. $g(x)$ is odd but not even.
$h(x)$ is neither even nor odd. $j(x)$ is neither even nor odd.
15. a. 1 mm per month
b. -1.5 mm per month
c. After 6 months, the marker will have moved 6 mm northward and about -4 mm westward ( 4 mm eastward). The marker has moved farthest in the northward direction.
d. If this trend continues, noticeable movement will continue only in the northward direction as the westward movement function appears to have an asymptote at $y=-4$.
