Introduction

Polynomials can be added and subtracted like real numbers. Adding and subtracting polynomials is a way to simplify expressions. It can also allow us to find a shorter way to represent a sum or difference.

Key Concepts

- A **monomial** is an expression with one term, consisting of a number, a variable, or the product of a number and variable(s).
- A **polynomial** is a monomial or a sum of monomials that contains variables, numeric quantities, or both. The variables of polynomials are raised to integer powers greater than 0. For example, $mn^4 - 6n$ and 12 are both polynomials.
- A **binomial** is a polynomial with two terms.

Key Concepts, *continued*

- Each part of a polynomial is called a term. A **term** is a number, a variable, or the product of a number and variable(s). For example, the second term in the polynomial $8p + 4r^3 + 9$ is $4r^3$.
- **Like terms** are terms that contain the same variables raised to the same power. Numeric quantities are like terms; for example, 1 and 3.4 are like terms. The terms $2x^3$ and $–4x^3$ are also like terms.

Key Concepts, *continued*

- Polynomials are typically written in descending order of the exponents; that is, from left to right, the term with the highest exponent is written first, followed by the term with the next highest exponent, and so on down to the term with the lowest exponent or no exponent.

Key Concepts, *continued*

- When there are two or more variables in a polynomial, the terms are written in descending order alphabetically. For example, $x^2 + x + y^2 + y + 12$ is written in descending order alphabetically.
- To add or subtract like terms containing a variable, use the Distributive Property to add or subtract the variable's coefficients. If a and b are coefficients and n is a positive integer, then $ax^n + bx^n = (a + b)x^n$.
- Subtraction can be represented in a similar way: $ax^n - bx^n = (a - b)x^n$.

Key Concepts, *continued*

- To add polynomials, combine like terms.
- Before subtracting one polynomial from another, rewrite the difference as a sum. For example, $(n + ax) - (m + bx)$ can be rewritten as the sum $(n + ax) + (-m - bx)$, as shown on the next slide.

Key Concepts, *continued*

$(n + ax) - (m + bx)$	Difference of two polynomials
$= (n + ax) + [-(m + bx)]$	Rewrite the difference as a sum.
$= (n + ax) + [-1(m + bx)]$	Rewrite $-(m + bx)$ as $-1(m + bx)$.
$= (n + ax) + [(-1)m + (-1)bx]$	Distribute -1 over $(m + bx)$.
$= (n + ax) + (-m - bx)$	Simplify.

- Therefore, subtracting two polynomials is the same as adding the opposite of the second polynomial.

Key Concepts, *continued*

- A system shows **closure**, or is closed, if the result of the operation is also in the system.
- For example, when an integer is added to an integer, the result is also an integer, so the set of integers is closed under addition.
- In the same way, the result of adding two polynomials is also a polynomial, so the system of polynomials is closed under addition.
- The result of subtracting one polynomial from another polynomial is also a polynomial; therefore, the system of polynomials is closed under subtraction.

Common Errors/Misconceptions

- finding a sum or difference of two terms with the same variable raised to different powers ($2x^3 + 5x^4 \neq 7x^3$)
- incorrectly adding or subtracting like terms by adding the exponents ($x^3 + x^3 \neq x^6$)
- incorrectly subtracting two polynomials by not rewriting the expression as a sum

Guided Practice

Example 1

Find the sum of $(4 + 3x) + (2 + x)$.

Guided Practice: Example 1, *continued*

1. Rewrite the sum so that like terms are together.

There are two numeric quantities, 4 and 2, and two terms that contain a variable, $3x$ and x. All the terms are positive.

$$(4 + 3x) + (2 + x)$$

$$= 4 + 2 + 3x + x$$

Guided Practice: Example 1, *continued*

2. Find the sum of any numeric quantities.

The numeric quantities in this example are 4 and 2.

$$4 + 2 + 3x + x$$

$$= 6 + 3x + x$$

Guided Practice: Example 1, *continued*

3. Find the sum of any terms with the same variable raised to the same power.

The two terms $3x$ and x both contain only the variable x raised to the first power.

$$6 + 3x + x$$

$$= 6 + 4x$$

The result of $(4 + 3x) + (2 + x)$ is $6 + 4x$.

Guided Practice: Example 1, *continued*

Guided Practice

Example 3

Find the difference of $(x^5 + 8) - (3x^5 + 5x)$.

Guided Practice: Example 3, *continued*

1. Rewrite the difference as a sum.

A difference can be written as a sum by adding the opposite of the second expression.

Simplify "– ($3x^5 + 5x$)" by distributing –1 and writing the polynomial as ($-3x^5 - 5x$).

$$(x^5 + 8) - (3x^5 + 5x)$$

$$= (x^5 + 8) + [-1(3x^5 + 5x)]$$

$$= (x^5 + 8) + (-3x^5 - 5x)$$

Guided Practice: Example 3, *continued*

2. Rewrite the sum so that any like terms are together.

Be sure to keep any negatives with the expression that follows, such as $–3x^5$.

$$(x^5 + 8) + (–3x^5 – 5x)$$

$$= x^5 + (–3x^5) + (–5x) + 8$$

Guided Practice: Example 3, *continued*

3. Find the sum of any terms with the same variable raised to the same power.

There are two terms with the variable x raised to the fifth power.

There is only one term with x raised to the first power, and only one numeric quantity.

The sum of the two terms with x^5 can be combined by adding their coefficients.

Guided Practice: Example 3, *continued*

$x^5 + (-3x^5) + (-5x) + 8$

$= -2x^5 - 5x + 8$

The result of $(x^5 + 8) - (3x^5 + 5x)$ is $-2x^5 - 5x + 8$.

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Guided Practice: Example 3, *continued*