Tiling Pools Learning Task

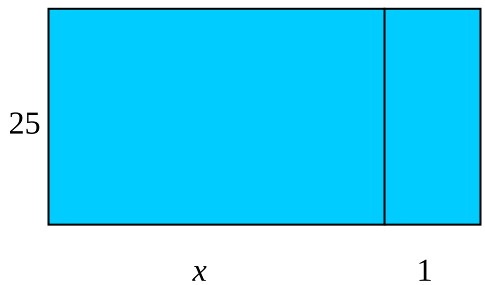
In this task, you will continue to explore how different ways of reasoning about a situation can lead to algebraic expressions that are different but equivalent to each other. We will use swimming pools as the context throughout this task.

In the figures below there are diagrams of swimming pools that have been divided into two sections. Swimming pools are often divided so that different sections are used for different purposes such as swimming laps, diving, area for small children, etc.

(a) For each pool, write two different but equivalent expressions for the total area.

(b) Explain how these diagrams and expressions illustrate the Distributive Property.

1.



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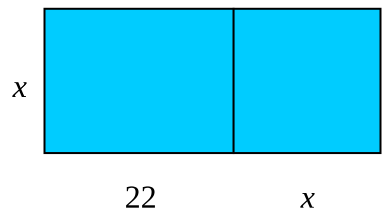
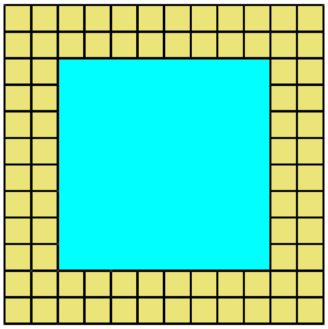


In-ground pools are usually surrounded by a waterproof surface such as concrete. Many homeowners have tile borders installed around the outside edges of their pools to make their pool area more attractive. Superior Pools specializes in custom pools for residential customers and often gets orders for square pools of different sizes. The diagram at the right shows a pool that is 8 feet on each side and is surrounded by two rows of square tiles. Superior Pools uses square tiles that are one foot on each side for all of its tile borders.



The manager at Superior Pools is responsible for telling the installation crew how many border tiles they need for each job and needs an equation for calculating the number of tiles needed for a square pool depending on the size of the pool. Let N represent the total number of tiles needed when the length of a side of the square pool is s feet and the border is two tiles wide.

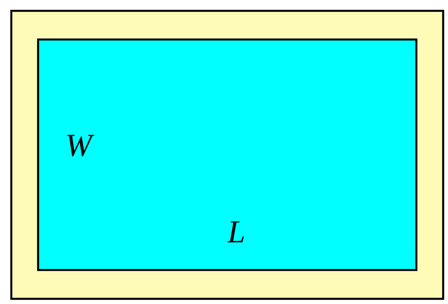
3. Write a formula in terms of the variable s that can be used to calculate N.

4. Write a different but equivalent formula that can be used to calculate N.

5. Give a geometric explanation of why the two different expressions in your formulas for the number of border tiles are equivalent expressions. Include diagrams.

6. Use the Commutative, Associative, and/or Distributive properties to show that your expressions for the number of border tiles are equivalent.

Some customers, who have pools installed by Superior Pools, want larger pools and choose a rectangular shape that is not a square. Many of these customers also choose to have tile borders that are 2 tiles wide.

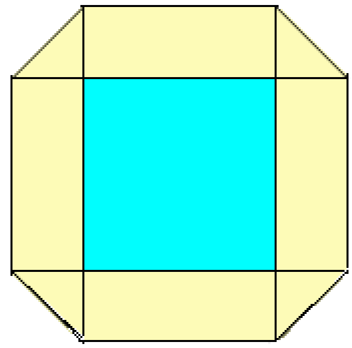


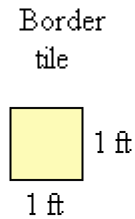
7. How many 1-foot square border tiles are needed to put a two-tile-wide border around a pool that is 12 feet wide and 30 feet long?

8. Write an equation for finding the number N of border tiles needed to put a two-tile-wide border around a pool that is L feet long and W feet wide. Explain, with diagrams, how you found your expression.

9. Explain why the area A of the tile border (in square feet) is the same number as the number of tiles that are needed for the border. Write an equation for finding the area A of the tile border using an expression that is different from but equivalent to the expression used in the equation for N given in answering question 8. Use algebraic properties to show that your expressions for A and N are equivalent.

A company that sells hot tubs creates a tile border for its products by placing 1-foot-square tiles along the edges of the tub and triangular tiles at the corners as shown. The triangular tiles are made by cutting the square tiles in half along a diagonal.





10. Suppose a hot tub has sides of length 6 feet. How many square tiles are needed for the border?

11. Write an equation for the number of square tiles N needed to create such a border on a hot tub that has sides that are s feet long.

12. Write a different but equivalent expression for the number of border tiles N. Explain why this expression is equivalent to the one given in your answer to question 11.

13. Below are three expressions that some students wrote for the number of tiles needed for the border of a square hot tub with sides s feet long.

(i) $4\left(\frac{s+s+1}{2}\right)$ (ii) $4\left(\frac{s}{2}+\frac{s}{4}\right)+2$ (iii) $(s+2)^2-4\left(\frac{1}{2}\right)-s^2$

(a) Use each expression to find the number of border tiles if $s = 0$.

(b) Do you think that the expressions are equivalent? Explain.

(c) Use each expression to find the number of tiles if $s = 10$. Does this result agree with your answer to part (b)? Explain.

(d) What can you say about testing specific values as a method for determining whether two different expressions are equivalent?

(e) Use algebraic properties to show the equivalence of those expressions in 11, 12, and 13 which are equivalent.